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AN INTERPRETATION OF THE DE VAUCOULEURS EMPIRICAL FORMULAE

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SUMMARY: In the framework of the axial symmetry and spheroidal geometry one looks for the interpretation of the de Vaucouleurs empirical formulae (describing the coordinate dependence of the surface density) for the discs and halos of spiral galaxies (also for the main bodies of elliptical galaxies). Using the general formula of the form $\rho(q) = A[q^b(a+q)^c]^{-1}$ (ρ is volume density), where A, a, b and c are constants (the exponents b and c are positive real numbers), one obtains satisfactory approximations for the solutions emanating from the empirical formulae describing the surface density.

1. INTRODUCTION

Gerard de Vaucouleurs, the distinguished American astronomer of French origin who recently passed away, studied intensively the images of external galaxies. Among others he examined the light distribution over the discs and halos of spiral galaxies and the "main bodies" of elliptical galaxies. The case of the exponential empirical laws describing the surface- brightness distribution in the main plane of the discs and in the tangential one, when the halos and elliptical galaxies are concerned, is well known.

One of the two exponential empirical laws - that concerning the "round" systems (halos of spiral galaxies and main bodies of ellipticals) - was discovered almost fifty years ago through the studying of elliptical galaxies (de Vaucouleurs, 1948). This formula was subjected to further tests by de Vaucouleurs himself, (e. g. de Vaucouleurs, 1977 and the references therein). As for the other exponential empirical law - that concerning the discs of S galaxies - though its form is due to de Vaucouleurs, indeed, (e. g. de Vaucouleurs, 1958), its extension and moderne application are in fact due to Freeman (1970). This law has in the meantime become generally accepted as a correct representation of the real situation (e. g. Binney and Tremaine, 1987 - p. 21). However, these two authors ascribe the empirical exponential law concerning the discs also to de Vaucouleurs by referring to the well-known Milky-Way model by de Vaucouleurs and Pence (1978).

The other exponential empirical law (concerning the round systems) has been also frequently assumed in the studies of various astronomers. However, though very simple in their mathematical expression, these two empirical laws are, nevertheless, not without difficulties, for example their transformation aimed at obtaining the corresponding formulae for the space density. In both cases there is no analytical solution. The experience of the recent 10-15 years in studying similar problems (Jaffe, 1983; Hernquist, 1990; Tremaine *et al.*, 1994) indicates the way which should be followed.

In the present paper a further generalisation of the family proposed by Tremaine *et al.* (1994) is carried out. In this manner one can find analytical formulae fitting the numerical solutions for the spatial density which result from the de Vaucouleurs formulae.

2. THEORETICAL BASE

The empirical formula describing the mass distributions within the discs, though later historically, will be written here before the other one due to its simplicity. It reads

$$\sigma(R) = \sigma(0)\exp(-R/R_d)\ , \quad R_d = \mathrm{const} \qquad (1)$$

where σ is the surface density, whereas R is the distance to the rotation axis or distance to the centre in the main plane (plane of symmetry).

In the case of a spheroidal system, where q is the semiaxis major of the equidensit spheroid and the x axis runs along the line of sight, we shall have

$$\sigma(\tilde{q}) = \sigma(0)\exp[-(\tilde{q}/R_h)^{1/4}]\ , \quad R_h = \mathrm{const}\ , \qquad (2)$$

$$\tilde{q} = [y^2 + (z/\epsilon)^2]^{1/2}, \quad q = (x^2 + \tilde{q}^2)^{1/2}\ .$$

Formula (2) is the de Vaucouleurs formula applied to the spheroidal systems (halos of spiral galaxies and main bodies of ellipticals); ϵ is the axial ratio, a constant as a matter of course.

Both formulae integrated to infinity, as easily seen, yield finite total masses.

In the present paper the spheroidal geometry is assumed also for the discs, however in this case the axial ratio is very small ($\epsilon \approx 0$). By introducing the reduced spatial density ρ^*, $\rho^* = \epsilon\rho$ the cases of both (1) and (2) can be treated mathematically in the same way. The integral equation solving the problem of finding the corresponding spatial density from the given surface one is

$$\rho^*(q) = -\pi^{-1}\int_q^\infty \frac{d\sigma}{dR}(R^2 - q^2)^{-1/2}dR\ . \qquad (3)$$

As easily seen, this case treats the projecting to the main plane, but the formula has the same form also for a plane perpendicular to the main one (formula (2) in the present paper) which is an advantage of introducing the reduced density.

It is well known that equation (3) for the cases of (1) and (2) admits no analytical solution. For example, Young (1976) published the tables corresponding to formula (2) and a plot of the obtained space density can be found in a paper of de Vaucouleurs, himself, (1977).

3. THE RESULTS

In the present paper equation (3) for both cases ((1) and (2)) is solved numerically. Among others, in both cases there are singularities at the centre. These singularities will be considered in more details below. The next step contains an attempt to find a formula fitting reasonably well the numerical solutions.

Recently Tremaine *et al.* (1994) considered a case of a model family. Their general formula is rewritten here in application to the spheroidal geometry, that is

$$\rho(q) \propto \frac{1}{q^b(a+q)^c}\ . \qquad (4)$$

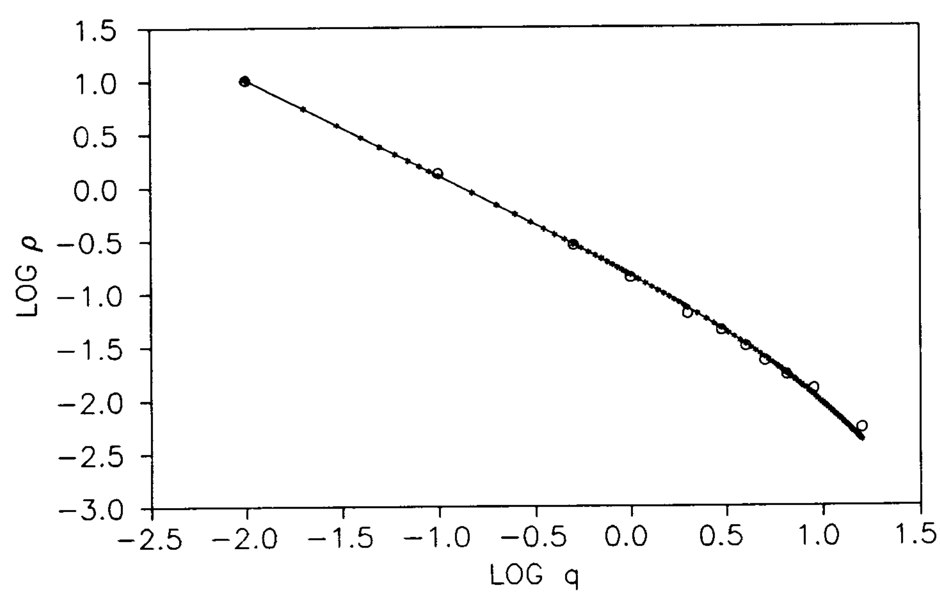

Fig. 1. *Agreement between numerical solution corresponding to (1) (circles) and formula (4) (solid line) - parameters:* $A = 333.867$ *(units see the text),* $a = 2.8R_d$, $b = 0.21$, $c = 5.05$.

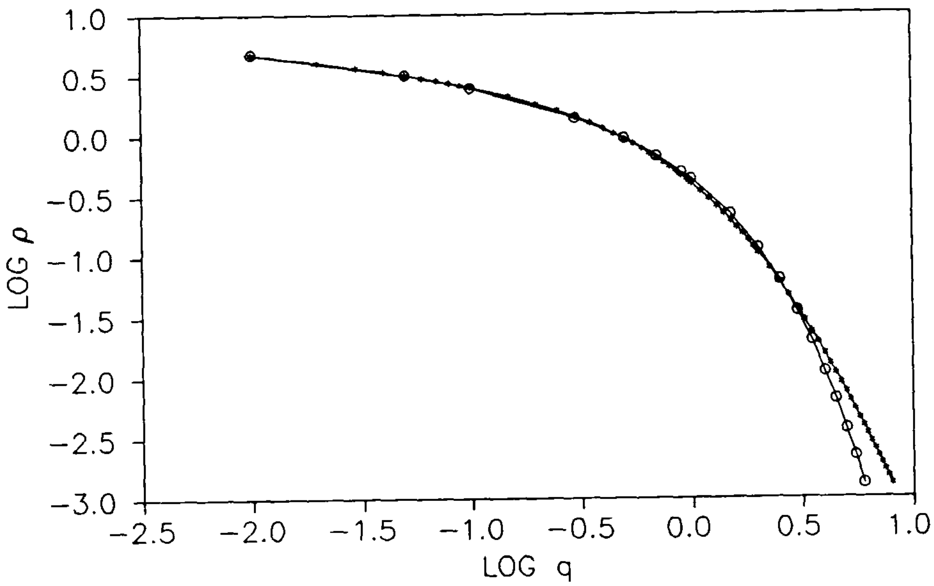

Fig. 2. *Agreement between numerical solution corresponding to (2) (circles) and formula (4) (solid line) - parameters:* $A = 938.093$ *(units see the text),* $a = 28.0R_h$*,* $b = 0.9$*,* $c = 2.6$*.*

a, b and c are, of course, constants. They fulfil the following conditions: $0 < b < 1$, $c > 0$, $b + c > 3$. The condition $b > 0$ enables to obtain a singularity at the centre (required by the numerical solutions as said above), the one $b < 1$ prevents to obtain analogous singularities in the resulting surface densities; as clearly seen formulae (1) and (2) contain no such singularities. Finally $b + c$ should be greater than 3 in order to have a finite total mass. These conditions are somewhat different from those from the paper of Tremaine *et al.* (1994). Namely, they include the marginal case ($b = 0$) with no singularity and require the sum $b + c$ to be always four. The latter circumstance is due to the fact that their formula was obtained by generalising the ones proposed earlier by Jaffe (1983) ($b = 2$) and Hernquist (1990) ($b = 1$). Thus Tremaine *et al.* (1994) do not require for b to be less than 1.

Therefore, formula (4) is assumed as a fitting one. What kind of fits is here obtained can be seen from Figures. Fig. 1 presents the case of the discs. The curve representing the space-density dependence is obtained with following parameters $A = 333.867$, $a = 2.8R_d$, $b = 0.21$, $c = 5.05$. The unit for the constant A corresponds to the density unit of $\sigma(0)/(\pi R_d)$. Fig. 2 presents the case of the halos (or main bodies of ellipticals). This time the values of the parameters are: $A = 938.093$, $a = 28.0R_h$, $b = 0.9$, $c = 2.6$; the unit for A, just as in the previous case, corresponds to a density unit of $\sigma(0)/(\pi R_h)$. As a general conclusion it may be said that the agreement in the far periphery ($q >> a$), in both cases, is less satisfactory. However, these parts of the system are unimportant since they contain only a tiny fraction of the total mass.

4. DISCUSSION AND CONCLUSIONS

The fits yield satisfactory agreements.

However, when speaking about the discs, one should say that their geometry need not be spheroidal, or more precisely, in stellar statistics their equidensit surfaces have not been always assumed to be spheroidal. In this connection there are many examples (e. g. Freeman, 1992). This circumstance is important. Namely, the spheroidal geometry does implicate the singularity in the space density which corresponds to (1). It is very easy to demonstrate that, for example, for the case of conic equidensit surfaces one obtains a surface-density function projected to the main plane identical to (1) assuming a space-density one of the form $\exp[-\alpha(R + |z|/\epsilon)]$ where both α and ϵ are constants. As clearly seen, this function has no singularity at the centre.

Therefore, we have the following question. Are spheroidal equidensit surfaces realistic indeed for the S-galaxies discs? This question is difficult to be answered because the concept of regular (mathematical) equidensit surfaces itself is, certainly, an approximation only. On the other hand, the spheroidal equidensit surfaces have been used, not once, for the purpose of describing such discs. The example of Schmidt's (1956) model for our own Galaxy is very well known.

Furthermore, in his own paper Hernquist (1990) claims that the formula proposed there yields a good approximation in the volume-density case for (2). However, the fact that his value of b (see (4)) is 1, exactly, gives rise, as already said above, to a singularity at $R = 0$ in the resulting surface density. Besides, the decrease of q^{-4} type in the outer parts predicted by his formula (again $b+c$ is 4 there) seems too strong in comparison with the numerical solution corresponding to (2). Therefore, one may conclude that the values for b and c used by him should be both corrected downwards in order to achieve a better agreement with the numerical solution. This is exactly what is obtained here. On the other hand, some empirical laws concerning the spatial distribution within the halos of (spiral) galaxies do suggest that their space density in the outer parts decreases following $q^{-3.5}$ rather than q^{-4} (e. g. Harris, 1976; Zinn, 1985).

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ЈЕДНО ТУМАЧЕЊЕ ДЕ ВОКУЛЕРОВИХ ЕМПИРИЈСКИХ ФОРМУЛА

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Оригинални научни рад

Тумачење де Вокулерових емпиријских формула (које описују зависност површинске густине од координата) за дискове и халое спиралних галаксија (такође за главна тела елиптичних галаксија) се тражи у оквиру обртне симетрије и сфероидне геометрије. Користећи општу формулу облика $\rho(q) = A[q^b(a+q)^c]^{-1}$ (ρ је запреминска густина), где су A, a, b и c константе (изложиоци b и c су позитивни реални бројеви), добијају се задовољавајуће апроксимације за решења која проистичу из емпиријских формула које описују површинску густину.