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SOME THREE-PARAMETER RELATIONS FOR PHYSICAL CHARACTERISTICS OF STARS

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SUMMARY: For homologous stars with $\mu(\rho, T)$ the L-$\mathcal{M}$-T_e relation is derived and tested against an observational material for the main-sequence stars. The transformations comprising the radius and the surface gravity are also given. The influence of the effective temperature on the 'mass-luminosity' and 'mass-radius' correlations is analysed.

1. INTRODUCTION

For given families of homologous stars under certain conditions it is valid $f_1(L, \mathcal{M}, R) =$ const where L, $\mathcal{M}$ and R are the luminosity, mass and radius of a star, respectively. On the other hand according to Stefan-Botzmann's law it is $f_2(L, R, T_e) =$ const where T_e is the effective temperature of the star. Therefore, from the two relations containing four unknowns (L, $\mathcal{M}$, R and T_e) any pair of them can be expressed through the another two. In other words in addition to f_1 and f_2 for the members of the chosen family there are also theoretical relations $f_3(L, \mathcal{M}, T_e) =$ const and $f_4(R, \mathcal{M}, T_e) =$ const.

If the homologous stars considered here are in a thermal equilibrium with the same nuclear-energy generation law, then from $L_{\rm nuc} = L$ it follows $f(R, \mathcal{M}) =$ const. Based on f_1, f_2 and f one can derive various two-parameter correlations: $L(\mathcal{M})$, i. e. the empirical 'mass-luminosity' relation following from f_1 and f is most widely used (e. g. Harris *et al.*, 1963; Mc Cluskey, Kondo, 1972; de Jager, 1980), as well as $L(T_e)$ – the empirical H-R diagram following from f_2 and f combined with $L(\mathcal{M})$.

Regarding that the observed stars are in a hydrostatic (quasihydrostatic) equilibrium, but need not be in a thermal one (gravitational contracting of the core begins already near the end of the main-sequence phase), in a more ample domain of changes of L, $\mathcal{M}$, R and T_e one should expect that three-parameter correlations have a higher real accuracy than the two-parameter ones. For example, in the case of the main-sequence stars due to the differences in the chemical composition and in the mechanisms of the transfer through the stellar interior, but also due to the different laws of energy production, it is impossible to derive a unique 'mass-luminosity' correlation except with the nonlinear fit of the observed values for $\mathcal{M}$ and L (for that case, about the calculation of the dynamical parallaxes and masses see Angelov, 1993). In this paper an empirical f_3 relation will be given and some consequences of it will be considered.

2. THE L-$\mathcal{M}$-T_e CORRELATION

Let it in a given family of homologous stars for the total pressure P, mean molecular weight μ and opacity κ be valid:

$$P = (k/\mu\beta)\rho T, \qquad \beta = P_{\rm gas}/P, \tag{1a}$$

$$\mu = \mu_0 \rho^{1-a} T^{1-b}, \tag{1b}$$

$$\kappa = \kappa_0 \rho^n T^{-s}, \tag{1c}$$

where k, β, μ_0, a, b, n, s are constants through the stellar interior and κ_0 depends on the chemical composition only. In view of $\rho(x) \propto \mathcal{M}R^{-3}$ and $P(x) \propto \mathcal{M}^2 R^{-4}$ at an arbitrary homologous point $x = r/R$ (r is the distance from the star centre), from (1a) and (1b) follows:

$$T(x) \propto \mathcal{M}^{(2-a)/b} R^{(3a-4)/b}\,. \tag{2}$$

It will be assumed that for the envelopes of the homologous stars within the family considered a radiative equilibrium with the same dependence $\kappa_0(x)$ is valid. In this case, based on (1c) and (2), for $L_{\rm rad}(x) = L(x)$ one obtains

$$L(x) \propto \mathcal{M}^{\xi} R^{\eta}, \tag{3}$$

where the constants ξ and η depend on a, b, n, s. The last relation (for $x \approx 1$) combined with $L \propto R^2 T_e^4$ yields:

$$L = {\rm const.}\, \mathcal{M}^{\alpha} T_e^{\beta}, \tag{4}$$

$$R = {\rm const.}\, \mathcal{M}^{\gamma} T_e^{-\delta}, \tag{5}$$

where

$$\alpha = 2\,\frac{(s+4)(2-a) - b(n+1)}{(s+4)(4-3a) - b(3n+2)}\,,$$

$$\beta = 4\,\frac{(s+4)(4-3a) - b(3n+4)}{(s+4)(4-3a) - b(3n+2)}\,, \tag{6}$$

$$\gamma = \frac{\alpha}{2}\,, \qquad \delta = \frac{4-\beta}{2}\,.$$

For (4) on the main sequence, with Popper's (1980) data one obtains:

$$\log L = -8.140 + 2.470 \log \mathcal{M} + 2.192 \log T_e\,. \tag{7}$$

In this case for the stellar radius one has

$$\log R = 3.454 + 1.235 \log \mathcal{M} - 0.904 \log T_e\,, \tag{8}$$

and for the surface gravity:

$$\log g = -2.470 - 1.470 \log \mathcal{M} + 1.808 \log T_e \tag{9}$$

and

$$\log L = -12.292 - 1.681 \log g + 5.232 \log T_e\,. \tag{10}$$

L, R and $\mathcal{M}$ in (7) – (10) are in solar units, g is in cm s^{-2}, whereas in deriving (8) and (9) are used the values $T_{e\odot} = 5780$ K and $\log g_\odot = 4.438$, respectively.

3. ANALYSIS AND CONCLUSION

The effective-temperature influence on the 'mass-luminosity' and 'mass-radius' relations along the main sequence is seen from (7) and (8). Regarding that changing T_e the value of the free term in (7) and (8) is also changed, the family of homologous stars is more homogeneous for the given value of T_e. If the lines T_e = const in the $\mathcal{M}$-L diagram (Fig. 1)

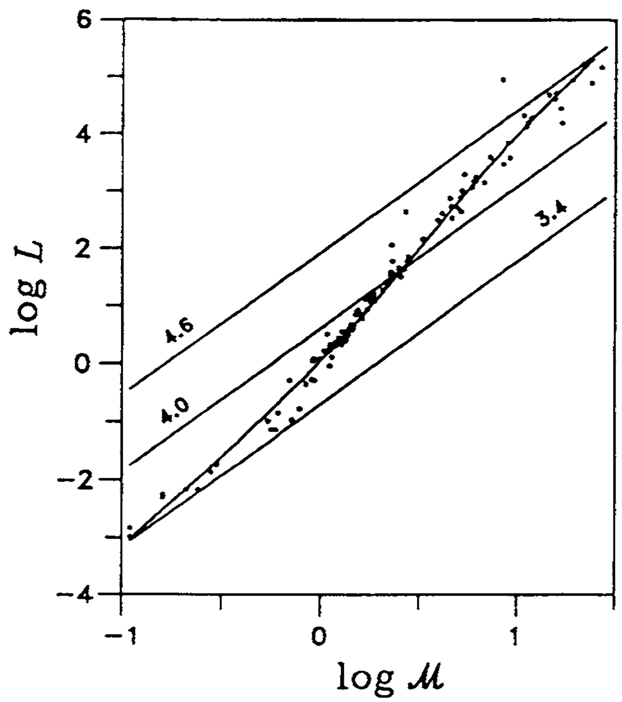

Fig. 1. Mass-luminosity diagram with Popper's (1980) data. The curve is a result of applying (7) to mean masses along the main sequence (L and $\mathcal{M}$ are in solar units). Three lines $\log T_e$ = const are presented.

were, approximatively, in the direction of the mean main-sequence line for the chosen mass interval, the T_e effect on the 'mass-luminosity' dependence in that $\Delta\mathcal{M}$ region would be negligible; an analogous conclusion is valid also for the $\mathcal{M}$-R diagram (Fig. 2).

Empirical relation (7) yields a better approximation for the 'mass-luminosity' diagram than a two-parameter L-$\mathcal{M}$ correlation. The same is true also for transformation (8) concerning the R-$\mathcal{M}$ correlation where the deviations are higher than in the previous case. This means that the value for R in a given mass and chemical composition is more strongly de-

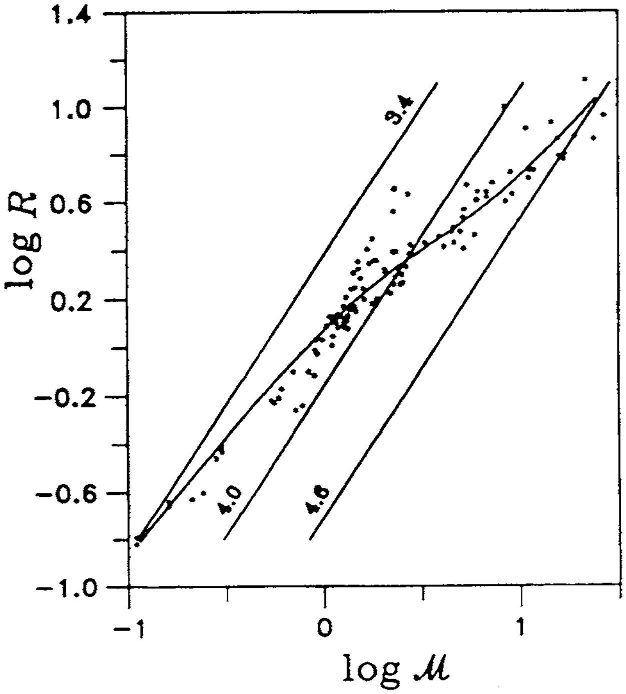

Fig. 2. Mass-radius diagram. Elements as in Fig. 1, but referred to (8).

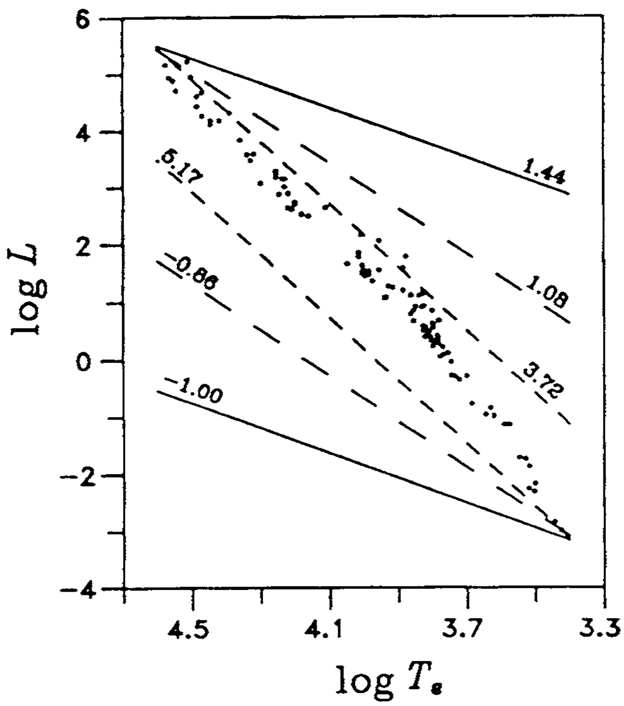

Fig. 3. The main-sequence region in double stars with mass measured. Two lines $\log \mathcal{M} = \mathrm{const}$ (full ones), and the corresponding lines $\log g = \mathrm{const}$ (- - -) according to (10) are contained; lines $\log R = \mathrm{const}$ (– –) are given for comparison (L, $\mathcal{M}$ and R are in solar units).

pendent (unlike L) of the thermal-equilibrium condition, and consequently, of the particular energy generation law (being in agreement with the theoretical results). On the other hand relations (7) and (10) in the MS region yield the lines $\mathcal{M} = \mathrm{const}$ and $g = \mathrm{const}$ (Fig. 3), whereas (9) transforms the $\mathcal{M}(T_e)$ relation into the $g(T_e)$ one (Fig. 4).

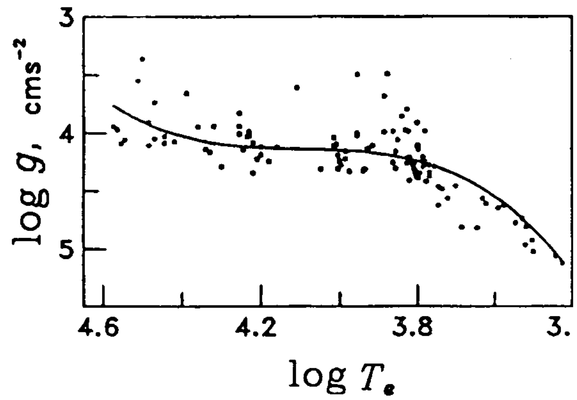

Fig. 4. The surface gravity depending on effective temperature on the main sequence – transformation $\mathcal{M}(T_e)$ into $g(T_e)$ according to (9). The field of g values with measured $\mathcal{M}$ and R is presented.

Based on (6), with the values for α and β from (7), or γ and δ from (8), for a and b given one can calculate the coefficients n and s (or vice versa):

$$n = 8\,\frac{\alpha - 1}{4 - \beta} - \frac{4(3\alpha - 1) - \beta}{2(4 - \beta)}\,a = 6.50 - 6.48\,a\,, \qquad (11)$$

$$s = -4 + \frac{4(3\alpha - 1) - \beta}{2(4 - \beta)}\,b = -4 + 6.48\,b\,. \qquad (12)$$

It is seen that a and b are practically stable in a wide interval of the values for n and s (e.g. for $n = 0$, $s = 0$ – Thomson's scattering, and $n = 1$, $s = 3.5$ – Kramers' law). Here according to (1b) the dependence of the mean molecular weight on the temperature appears as dominant, whereas in the case of a constant chemical composition ($a = 1$, $b = 1$) the coefficients in (1c) are $n \approx 0.02$, $s \approx 2.5$.

It is, however, clear that the assumptions contained in relation (4) are not valid for the entire main sequence (though the formal accuracy of the corresponding correlation is satisfactory – the relative errors of coefficients in (7) are less than 10 %). Namely, for $T_e < 7000 - 7500\,\mathrm{K}$ (approximately for $\mathcal{M} < 1.7\,\mathcal{M}_\odot$) the convection in the star envelopes along the main sequence becomes effective as the transfer mechanism (see Cox and Giuli, 1968, p.609).

There is also the possibility of a conditional dividing of the upper MS branch into two parts with respect to the dominant absorption processes (in the part near its end, i. e. in the envelopes of massive stars, Thomson's scattering prevails). However, a main-sequence division into two, i. e. three regions, does not yield expected results.

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НЕКЕ ТРОПАРАМЕТАРСКЕ РЕЛАЦИЈЕ ФИЗИЧКИХ КАРАКТЕРИСТИКА ЗВЕЗДА

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За хомологне звезде са $\mu(\rho, T)$ изводи се релација L-$\mathcal{M}$-T_e и тестира на посматрачком материјалу за звезде главног низа. Такође, дају се трансформације за радијус и гравитационо убрзање на површини звезде. Анализира се утицај ефективне температуре на корелације 'маса-сјај' и 'маса-радијус'.