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Review paper

DOUBLY-EXCITED ATOMS AND THE LINE BROADENING*

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SUMMARY: Atomic systems with doubly-excited electrons have been the objects of extensive studies in the last fifteen years. These autoionizing states may be long-lived metastable atoms, with a number of properties peculiar to those systems where interelectron correlations play crucial role, and the independent-particle picture fails even as a zero-order approximation.

We shall expound principal mechanisms for forming such states, quote essential features of the two-electron excited states and the most common ways for their decays. Particular attention will be paid to the methods for evaluating energy spectra, especially quantum mechanical and semiclassical approaches for calculating line positions and widths.

Possible modes of radiative transitions will be enumerated and a number of mechanisms of line broadening shall be discussed. Some prospective developments in the line broadening theory will be outlined.

1. INTRODUCTION

Just as the classical dynamics and Quantum mechanics (particularly its proto-form in the Old quantum theory of Bohr) were more or less confined to the integrable systems (at least as far as the analytical theory was concerned), the theory of atomic structure was until recently restricted to the essentially single-electron model. The latter was the cornerstone of all further generalizations to many-electron atoms, and all elaborations concerned with more subtle effects, in particular those related to the *interelectron correlations*, were essentially of an approximative nature. These approximations have been used, however, efficiently in calculating atomic structures (and processes, like collisional ones), and Quantum mechanics has proved to be reliable and powerful tool in atomic physics, whose efficiency has been restricted only by our mathematical (in)abilities to deal with equations with many variables.

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The classical models of atoms were abandoned soon after Schrödinger and Heisenberg formulated Quantum mechanics, and until early eighties no serious attempts to describe few-electron atoms (semi)-classically were made. On the other hand, Schrödinger equation turned out to be easily adaptable to many-electron systems, and extensive calculations of atomic states and processes have been done since the appearing of new quantum theory. But not all atomic states (and/or processes) turn out equally easy (or difficult) to deal quantum mechanically. This is particularly true when highly excited states are involved, for the mixing and interference of many closely related discrete atomic states make analytical calculations very cumbersome and computing times prohibitive. It is these excited states that are present in the line broadening phenomena and correct descriptions of the levels of transitions (particularly the upper ones) cannot be overestimated.

As a rule, only single-electron transitions have been considered so far when the line broadening are investigated. This choice has been dictated both by the practical importance of these processes and by theoretical difficulties involved in describing few-electron transitions. On the other hand, recent advances in the theory of few-electron excitations and experimental achievements in the spectroscopy studies of processes when not only single-electron transitions occur, have made the problem of multiply excited (in particular doubly-excited) states both feasible and relevant. Which consequences these studies might have to the line-broadening physics? In this lecture we shall try to enumerate some relevant features of these many-electron processes and estimate possible developments and importance of the studies of two-electron excited states to the plasma spectroscopy in general. Because of the lack of actual calculations, the emphasis will be on the phenomenological and conceptual aspects of the problem. This will dictate, consequently, a more qualitative level of exposition, with an attempt to present a systematic set up for the envisaged arena of the experimental and theoretical endeavors in the field.

2. PHENOMENOLOGY OF DOUBLE EXCITATIONS

Doubly-excited atomic states can be formed in many ways. We shall enumerate some of the most important mechanisms for producing atoms with two (or more) electrons occupying energy levels higher than their respective ground states energies.

(a) Direct collisional excitations, by charged projectiles, or photons (photo-double-excitation (Camus, 1994). It was the latter mechanism that has been observed experimentally first (Madden and Codling, 1963) and this experiment triggered an avalanche of doubly-excited atoms studies, both experimentally and theoretically (Fano, 1983). Excitations by charged projectiles (Pederson and Hvelplund, 1989) have revealed the prominent role of electron-correlation effects, which have been the subject of theoretical quantum mechanical investigations (McGuire and Straton, 1991; Martin and Salin, 1995) (see also recent semiclassical calculations (Slim *et al.* 1995)). These excitations populate predominately *intrashell states*, that is both highly-excited electrons have approximately the same principal quantum numbers (Domke *et al.* 1991).

(b) Resonant scattering by electron impact, on target as neutral atoms or positive ions. Two types of resonances may appear : (i) the impact electron is captured by the target in its ground state (so-called shape resonances); (ii) Feshbach resonant state, when a target electron is excited, and the so-called *compound state*, with two excited electrons, is formed. It is the latter case which ir relevant to our issue. There is an abundant literature on the resonant scattering and we mention just two recent calculations by Fon *et al.* (1989, 1994). We note here that the resonant mechanism tends to populate low-lying double excited states, with different principal electron quantum numbers (*intershell states*).

(c) Dielectronic recombination, when an impact electron is captured by a positive ion, with the accompanying excitation of one of target electrons. This is, in fact, a subclass of the processes from b(ii) (Bates Dalgarno, 1962).

(d) Double-electron exchange, which takes place in a multiply-charged ion impinging on a neutral (or ionized) target or on a surface, with two electrons captured from the target, occupying excited states around the projectile ion (Barat and Roncin, 1992; Belkić, 1993; Stolterfoht, 1994). Contrary to (a), high-lying states formed in this way are mainly of the *intershell* type, *ie* have different principal quantum numbers (Chen and Lin, 1993).

3. CLASSIFICATION SCHEMES

Most crudely, one distinguishes two large classes of doubly-excited states:

(a) Planetary atoms

As the name implies, electron occupy portions of the real space with very different average values of their positions relative to the nucleus. In classical terms, their orbits may resemble those of a solar system, like our own.

(b) (Quasi)equivalent electrons

In this case all physical properties of single-election states acquire approximately the same values. Speaking in terms of the single-electron quantum numbers (which need not, and usually are not an adequate description, as we shall see immediately), these have similar numerical values, in particular principal quantum numbers may be identical (so-called intrashell resonances).

3.1 Semiclassical theory

However, for a proper classification one must resort to theoretical models, which are able to make fine distinctions between states, introducing further subclasses, subsubclasses *etc.* These refinements are best realized within semiclassical models, based on particular classical electron configurations. We shall not, however, dwell on these divisions, bur refer interested readers to a more specialized literature. Nevertheless, for understanding the nature and peculiarities of these states, we need some semiclassical description, at least in order to introduce an appropriate way of state designation.

Unfortunately, there is no unique labeling of the (semi)quantum states. We shall, first restrict ourselves to *planar models*, which have been most intensively studied so far. They are at the same time most important, for the $L = 0$ (zero total angular momentum) case belongs to the configuration with both electrons moving in the same plane. Further, the models developed are strictly applicable to the two-electron systems, *ie* to the helium-like atoms. Doubly-excited atoms with more than two electrons may be treated within the same semiclassical theory, but the residual electronic core introduces inevitably further inaccuracies in the theory, which may be, however, dealt with efficiently, if Rydberg atoms are considered (average individual momenta of either of excited electrons being large).

3.2 Correlation effects and quantum numbers

Correlations in electrons moving within doubly excited states play much more prominent role than in ground-state case. The effect of interelectron correlations varies from one particular underlying classical configuration to the other. Planetary atoms exhibit very weak correlation between inner and outer electron motion, for the active electrons have periods very different from those close to the nucleus. On the contrary, equivalent-electron kinematics is endowed with highly correlated motion, and the electron pair appears as a single quantity, similar to Cooper pairs in superconductivity. Since the two-electron systems are nonintegrable, various systems of *collective coordinates* are used, and corresponding collective quantum numbers come out from these descriptions. It is for these configurations that the most effective designation schemes have been proposed. Of these, the most widely used is the set of quantum numbers proposed by Lin, as a generalization of Herrick's classification, based on SO_4 symmetry group of two-electron (independent-particle) system. Since quantum number designations play important role in dealing with two-electron atoms, we shall dwell upon the subject in somewhat more detail.

The standard independent-particle state designations $n_i\ell_i m_{\ell_i}$ fail when the motion is highly correlated, for the corresponding single-particle quantities appear not constant even approximately. The only exact integrals of the system are the energy (E), angular momentum ($\mathbf{L}$), spin S and parity π. Then, the system is designated by ($LS\pi$). The energy and other quantities depend on the internal structure of the system, which must be specified by further quantum numbers. They must necessarily be of approximate nature and how much good they are depend on our choice of suitable coordinate systems adopted. The latter, in its turn, depend on the type of (expected) electronic configuration and on a particular model selected within this class of configurations.

Although quantum mechanical description is an ultimate goal in the atomic structure theory, in many situations semiclassical approach appears both legitimate and efficient. This is particularly so in the case of highly doubly-excited states, where the *correspondence principle* holds. Moreover, the overall experience shows that even for moderately excited states semiclassical theory yields reasonable results and even some features of the ground states may be fairly adequately described by the semiclassical picture. Another important feature of the approach based on the notion of classical trajectory, is its physical transparency, which quantum mechanical description usually lacks. So, it comes as no surprise that in this new area of atomic structure theory semiclassical approach has been extensively used and we shall start by the semiclassical picture in defining relevant atomic features.

We shall make use of two principal sets of coordinates. The first is the so-called *hyperspherical coordinate system*, defined as (Fano, 1983),

$$\begin{aligned} \theta_{12} &= \angle(\vec{r}_1, \vec{r}_2) \quad - \quad \text{mutual angle,} \\ \alpha &= \arctan(r_2/r_1) \quad - \quad \text{hyperangle,} \\ R &= (r_1^2 + r_2^2)^{1/2} \quad - \quad \text{hyperradius.} \end{aligned} \tag{1}$$

In many physical systems it appears that some coordinates change slowly in time (slow variable), and some fast (fast variable). The first variable are, obviously, convenient to which to ascribe quantum numbers (adiabatic variable). If one denotes electron positions with respect to the nucleus (residing at the

origin) $\mathbf{r}_1, \mathbf{r}_2$, the relative vector $\mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2$, and position of the electron-pair centre-of-mass by $\mathbf{r}$, then neglecting spin-orbit interaction, one can write the three-body wavefunction as

$$\Psi = \psi^{S,t}_{LM}(\mathbf{r}, \mathbf{R})\chi_{SM_S}\xi_{sM_t}. \qquad (2)$$

where ψ is the spatial wavefunction, χ and ξ spin functions of the electron pair and the remaining nucleus respectively and $t = 0, 1$ for *gerade, ungerade* symmetry. Treating the two-electron atoms as rigid-body systems, with Θ, Ψ and ϕ Euler angles which determine orientation of the body-fixed reference frame relative to the space-fixed one, then the two body-fixed coordinate of $\mathbf{r}$ are the so-called prolate spheroidal coordinates

$$\lambda = (r_1 + r_2)/R, \qquad \mu = (r_1 - r_2)/R \qquad (3)$$

These coordinates, together with the azimuthal angle ϕ, are well suited to describe the two-electron systems in terms of the molecular-orbital states, which will prove to be well suited in their turns to determine the corresponding quantum numbers, as we shall see later on.

The Herrick-Lin (HL) notation $(NKTn)^A$ (Herrick, 1975; Lin, 1984) is based on the properties of Keplerian (independent-electron) orbits. The inner electron has the principal quantum number $(N \leq n)$, where n is the principal quantum number of the outer electron. Then, K is defined as $K \approx < r_1 cos\theta_{12} >$, with the meaning that the closest integer to the numerical value of the averaging $<>$ over period is taken. The other quantum number T is defined as T^2 to be proportional to the average value of the square of vector $-\mathbf{l}_1\hat{\mathbf{r}}_2$, where $\mathbf{l}_1$ denotes the first electron angular-momentum vector. These two quantities (and corresponding quantum numbers) refer to average (global) properties of underlying classical configurations (integral features). Third quantum number A characterizes the type of kinematics involved. It appears a measure of *radial correlations*. Roughly speaking, if electrons oscillate *in-phase* (symmetric stretch, in molecular dynamics terms), then $A = +$, for *out-of-phase* (antisymmetric stretch) motion $A = -$, and for a mixed case, when there is no preference as to the phase correlation, one has $A = 0$. Referring to the properties of corresponding wavefunctions, all $^1S^e$ states have $A = +$, and all $^3S^e$ have $A = -$. As for $A = 0$ states, they appear similar to singly-excited states. In terms of hyperspherical coordinates, $A = +$ states have an antinode at $\alpha = \pi/4$, whereas $A = -$ states have a node there. Finally, for all states which correspond to planar semiclassical configurations, such as $\mathbf{L} = 0$ states, one has $T = 0$.

T and K measure the extent of angular correlations and for a given set of L, N assume the following values ;

$$T = 0, 1, ..., min(L, N-1) \qquad (4)$$

$$\pm K = N - T - 1, N - T - 3, ..., 1(0) \qquad (5)$$

Further, for states with parity $\pi = (-1)^{L+1}, T = 0$ (planar configurations) is not allowed. Thus, for $N = 3$ states, one has state labeling shown in Table 1.

Table 1. Herrick-Lin designations of doubly-excited states for N = 3 (Lin, 1984)

$^{2S+1}L^{\pi}$	(K,T)					
$^{1,3}S^e$	(2,0)	(0,0)	(-2,0)			
$^{1,3}P^e$	(2,0)	(1,1)	(0,0)	(-1,1)	(-2,0)	
$L \geq 2$	(2,0)	(1,1)	(0,2)	(0,0)	(-1,1)	(-2,0)

Treating the two-electron atoms as two-centre problem one finds that the latter appears separable in (λ, μ, ϕ) coordinate system, with corresponding quantum numbers $(n_\lambda, n_\mu, n_\phi \equiv m)$. In many situations these quantum numbers turn out to represent better doubly-excited states, *ie* they are better quantum numbers than for other choices. They are related to other quantum numbers choices and in Table 2 connections with *parabolic* (separated-atom limit : $R \to \infty$) and *spherical* (united-atom limit : $R \to 0$) quantum numbers are shown, together with the corresponding Herrick-Lin designation $(A = (-1)^{n_\mu})$.

Table 2. Quantum-states designations for two-centre-Coulomb systems, within the separated-atom (SA), molecular-orbital (MO), united-atom (UA) and Herrick-Lin (HL) schemes (from Rost and Briggs (1991))

UA	MO	SA		HL
$(n\ell m)$	$(n_\lambda n_\mu m_\phi)$	$(n_1 n_2 m_\phi)^A$	N	$(K,T)^A$
$1s\sigma_g$	(000)	$(000)^+$	1	$(00)^+$
$2p\sigma_u$	(010)	$(000)^-$	1	$(00)^-$
$3d\sigma_g$	(020)	$(010)^+$	2	$(10)^+$
$4f\sigma_u$	(030)	$(010)^-$	2	$(10)^-$
$2p\pi_u$	(001)	$(001)^+$	2	$(01)^+$
$3d\pi_g$	(011)	$(001)^-$	2	$(01)^-$
$2s\sigma_g$	(100)	$(100)^+$	2	$(10)^+$
$3p\sigma_u$	(110)	$(100)^-$	2	$(10)^-$

In the literature other quantum numbers appear as well, and in equations (6) we quote some of the most important mutual relations (Rost and Briggs, 1991)

$$\begin{aligned} T &= m(\equiv n_\phi), \\ A &= (-1)^{n_\mu}, \\ K &= n_2 - n_1 = [n_\mu/2] - n_\lambda, \\ \nu_2 &= 2n_1 + m = 2n_\lambda + m, \\ d &= n_2 + m = [n_\mu/2] + m. \end{aligned} \tag{6}$$

4. THE ENERGY SPECTRA

Location of the energy level positions of doubly-excited atoms has been the subject of intensive studies for the last two decades. Two main stages in studying energy levels have been a phenomenological approach and *ab initio* level calculations.

4.1 Rydberg formulae

For a planetary model state the simple Rydberg formula of the form (atomic units are used throughout, unless otherwise stated)

$$E_n = I_1 - \frac{Z'}{(n-\delta)^2} \tag{7}$$

I_1 is the single-ionization potential, and δ is the quantum defect (presumably independent of n and only weakly depending on ℓ), gave reasonable values for the line position. For the symmetrically excited, intrashell states, a modified, so-called double-Rydberg formula (Read, 1977) was employed (see also Kazansky and Ostrovsky, 1994)

$$E_n = I_1 - \frac{(Z'-\sigma)}{(n-\delta)^2} \tag{8}$$

where σ is the screening constant, usually not much different from 0.25 (symmetrical configurations : $\mathbf{r}_1 = -\mathbf{r}_2$). This simple formula provided a fair match to experimental peaks in the excitation functions (*eg* Cvejanović and Read, 1974), but for more accurate evaluations other more refined theories are needed (Ezra *et al.* 1991). Besides, formulae like (8) do not provide energy widths. Nevertheless, some elaborate phenomenological formulas do provide good fits to the observed resonance positions. Here we quote two-electron formula due to Sadeghpour and Green (1990)

$$E(m,n) = -\frac{1}{2n^2} - exp\left[\frac{2\pi(n-m)}{a_{n,n-2,1}}\right] \times \left[\frac{(1-\sigma)^2}{(n-\mu)^2} - \frac{1}{2n^2}\right] \tag{9}$$

where

$$a_{n,n-2,1} \approx 3n^2 - 23n/3 + 2/3n + 1 \tag{10}$$

and

$$\alpha_{n,n-2,1} = \sqrt{a_{n,n-2,1} - 1/4}, \quad m = n \tag{11}$$

refers to intrashell (Wannier) states and $m = n + 1, n + 2, ...$ gives intershell (or dipole) states.

4.2 Accurate energy levels

Although quantum mechanically doubly excited spectra can be accurately evaluated, because the energy levels become very dense as the ionization limit is approached and many states must be accounted for, calculations become progressively more involved and consequently prohibitive computing time appears required. Quantum mechanical calculations, nevertheless have been carried out, but here they will serve us mainly as reference data for simpler and physically more transparent semiclassical calculations. The latter are based on particular classical underlying configurations and semiclassical formulae for the energy spectra have been developed for each particular semiclassical model (*eg* Grujić and Simonović, to be published). We shall quote some of them, mainly for the purpose of illustration.

4.2.1 Frozen-planet model

This is a very peculiar two-electron configuration, confirmed both experimentally (Camus *et al.* 1989) and theoretically *eg* (Richter *et al.* 1992). In figure 1 we show the underlying classical configurations for two states. In figure 1(a) the outer electron is practically located at a point (therefore *frozen-planet* term), whereas in figure 1(b) excitations along angular and radial modes are present.

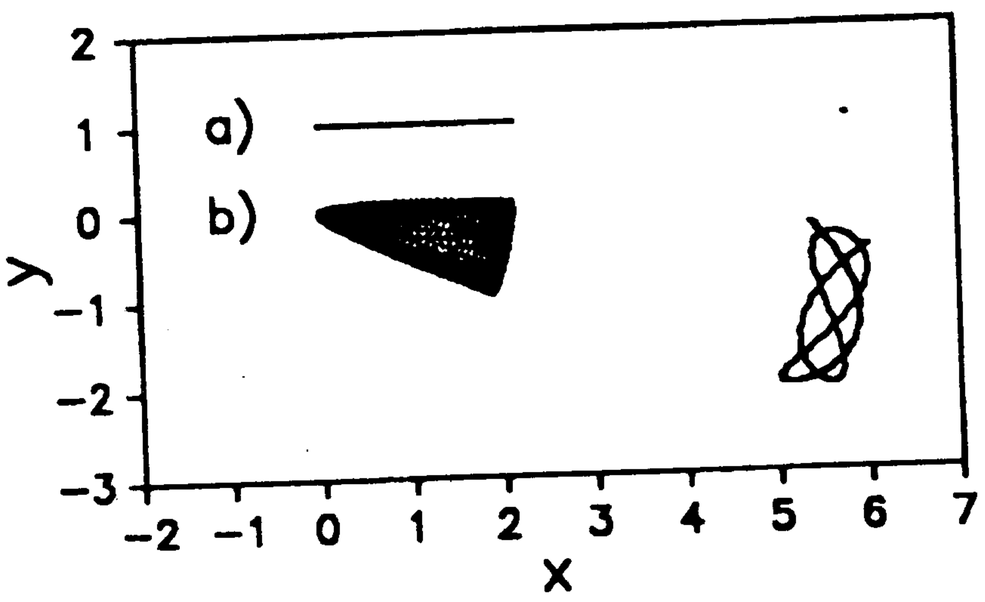

Figure 1. Frozen-planet model for two-electron atoms. (a) The outer electron is trapped into a small configuration-space region, while the inner electron circles around the nucleus (Z = 2); (b) bounded motion of the outer electron around the equilibrium point from (a). Both configurations appear (classically) stable against the autoionization (see text).

The semiclassical energy levels are given by the so-called triple-Rydberg formula (Richter and Wintgen, 1991)

$$E_{nkl} = \frac{-S^2}{[(n+\frac{1}{2}) + (l+\frac{1}{2})\gamma_R + (2k+1)\gamma_\theta]^2} \qquad (12)$$

where S is the classical action, γ are the so-called *winding numbers*, and n, k, l are semiclassical quantum numbers, describing excitations along the orbit(n), and perpendicular to the orbit (k, l). In Table 3 we give some numerical values for $^1S^e$ states of doubly excited helium, without perpendicular excitations (Wintgen *et al.* 1994), together with the corresponding quantum mechanical results (note that the semiclassical theory provides energy positions only).

Table 3. Semiclassical energies and quantum mechanical energies and total decay widths (against autoionization) for the frozen-planet states of $^1S^e$ state of helium principal series, with perpendicular excitations suppressed ($k = l = 0$) (Wintgen *et al.* 1994)

n	$-E_{scl}$	$-E_{n00}$	$\Gamma/2(\times 10^4)$
2	0.24792	0.2573716	0.105640
3	0.13935	0.1410641	0.117392
4	0.089145	0.0895708	0.020241
5	0.061887	0.0620535	0.05602
15	0.0086947	0.0086970	0.000014

As can be seen from Table 3 semiclassical formula predicts well the energy positions, with increasing accuracy as the principal quantum number increases. The energy width decreases rapidly as the ionization limit is approached, indicating an important property of doubly excited states - they are more and more stable as the excitation degree is larger. Doubly excited states are subject to autoionization and in the case at hand the probability for decay process is decreasing exponentially. In this particular case there is no potential barrier, but a so-called *dynamical tunneling* takes place instead. A state localized on a torus near the periodic orbit may decay by tunneling into the region outside the stability island, which allows for an autoionization (one electron is de-excited and the other moves to infinity).

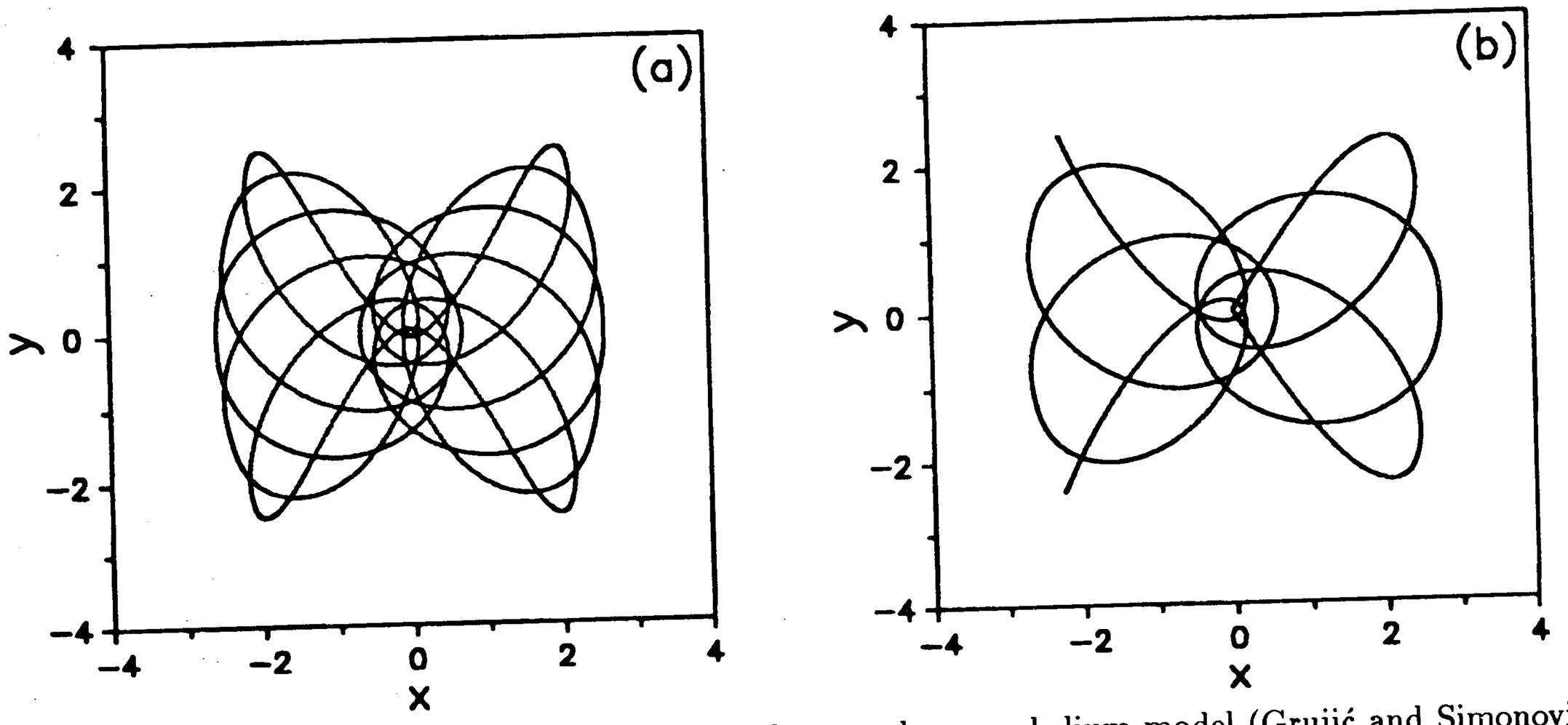

Figure 2. Two typical classical configurations of the asynchronous helium model (Grujić and Simonović, 1995).

Frozen planet configuration belongs to the class of regular motion, with periodic classical orbits (motion on a torus). There exist other (semi)regular models, but we shall not dwell on them and direct interesting readers to review articles, like Wintgen *et al.* (1992); Grujić and Simonović (to be published). We show only in figure 2 a classical configuration of an intrashell doubly-excited state (Grujić and Simonović, 1995), for the sake of completeness. The electrons move out-of-phase, along complicated trajectories, with the total angular momentum $L = 0$..

Another important class of semiclassical configurations is that of irregular (chaotic) underlying classical kinematics and the problem of quantizing them appears a much more involved task, though some significant advances have been made (Wintgen *al.* 1992; Wintgen *et al.* 1994).

4.2.2 Quantum mechanical calculations

Various methods in calculating energy levels have been employed and we shall quote some results for the most frequently ${}^1S^e$ helium states. In Table 4 we show some of the results due to Ho (1986) (complex-coordinate rotation), Oza (1986) (algebraic close-coupling) and Müller *et al.* (1994) (stabilization and complex rotation methods). As can be seen from Table 4 agreement between various methods is satisfactory.

There has been some controversy about level-widths behaviour in the asymptotic region (*ie* for large $N = n$ values), which can be written in the form

$$\Gamma \sim N^{-\nu} \tag{13}$$

Majority of calculations provide $\nu \approx 6$ (see Nicolaides and Komninos, 1990 and references therein), whereas estimates based on near-threshold behaviour and on purely classical arguments (Cvejanović *et al.* 1990) suggest $\nu \approx 3$. Also, still another estimate is due to Watanabe (1987)

$$\Gamma \sim N^{-3} lnN \tag{14}$$

Experimental evidence yields $\Gamma \sim (N^*)^{-3}$ (Domke *et al.* 1991), what has been supported by recent calculations by Ostrovsky and Prudov (1993). But apart from the lack of agreement on the exact form of the asymptotic formula, all evidence points to the high stability of the highly double-excited states (see Tables 3,4), what appears contrary to our intuition, considering the corresponding increase of the number of lower states, to which a particular state may decay (see Table 5 in Richter *et al.* (1992)).

Table 4. Doubly-excited ${}^1S^e$ states of helium due to Ho (1986), Oza (1986) and Müller *et al.* (1994). Both energy positions E_r and widths $\Gamma/2$ are in Rydbergs (1R = au/2 = 13.6 eV).

K	T	N	n	$-E_r$	$\Gamma/2$	$-E_r$	$\Gamma/2$
				Ho		Oza	
1	0	2	2	1.555736	0.00453	1.5556	0.00458
1	0	2	3	1.17979	0.00135	1.17973	0.00138
1	0	2	4	1.08975	0.00045	1.08974	0.00049
-1	0	2	2	1.234855	0.0002156	1.241032	0.000231
-1	0	2	3	1.096171	0.000078	1.095753	0.0000827
-1	0	2	4	1.05542	0.0005	1.05525	0.000052
				Müller *et al.*			
				stabilization		complex rotation	
-1	0	4	4	0.16826	0.00212	0.16826	0.00217
3	0	4	5	0.16574	0.00122	0.16573	0.00121
5	0	6	9	0.067804	0.000121	0.67803	0.000118
-3	0	6	6	0.067932	0.000083	0.067931	0.000083
8	0	9	9	0.040474	0.000019	0.040474	0.000019
7	0	8	8	0.051389	0.000006	0.051389	0.000006

5. NON-RADIATIVE AND RADIATIVE TRANSITIONS

Unlike single-electron excitations which finish by radiative de-excitations, doubly-excited states have much more freedom to de-excite. Transitions to ground (or other lower) state can be divided into two broad classes - radiative and non-radiative de-excitations.

Strictly speaking only single-electron atoms (more precisely - only two-body systems) may have single-electron transitions. The usual treatments of "singly-excited" atoms is based on tacit assumption that the independent-particle approximation is valid. The latter means that one may use single-particle quantum numbers, as designations like $1s^2 2s^2 2p...$ implies. Neglecting spin-orbit and other relativistic effects, only transitions between states described by sets like $^{2S+1}L^{\pi}_{J}$ should be considered. Nevertheless, approximations involved in standard designations of the "electron configurations", like the above-mentioned one, provide usually reasonable results. The underlying assumption is that the residual-core electrons are strongly correlated (in fact, absolutely correlated), whereas the excited electron interacts with the residue *via* a sort of mean-field approximation. Of course, accurate theoretical methods do account for the approximate nature of the electron-configuration picture, by incorporating many different states (like *configuration-interaction* method) *etc.*

As mentioned before, correlations play even more crucial role in doubly-excited atoms (see *eg* Camus *et al.* 1989). Not only that the electron-electron interaction cannot be neglected, but many of the (semi)classical models owe to this its very existence. The frozen-planet configuration, for instance, could not exist without strictly correlated motions of the inner and outer electrons. These models (the so-called Langmuir oscillatory model belongs to this class) appear thus of essentially non-perturbative nature (Grujić and Simonović, to be published), and the electron pair turns out to be an analogue of the single electron in the independent-particle theoretical schemes. As a consequence of this strong-correlation effects, single-electron transitions of doubly-excited atoms are not possible. Both excited electrons participate in any kind of the configuration change and thus in any transition involved. Non-radiative and radiative transitions are two competing processes in doubly-excited atoms, just they are in some single-electron excitations from inner shells (like Auger processes (Åberg, 1992; Stolterfoht, 1994).

5.1 Non-radiative transitions

Non-radiative changes of configurations may be divided into two kinds of transitions : *on-energy-shell* and *off-energy-shell* ones.

5.1.1 Off-energy-shell transitions

They are induced in collisional processes. In fact, only those involving ground state of the target are observed so far (Pederson, 1990), whereas other doubly-excited states might be considered as transient configurations only. One defines so-called *static* and *scattering correlations*, depending whether these appear prominent in the asymptotic zone or within the inner region during the very collision process (Pederson, 1990). The latter effects are present particularly at low scattering energies, when there is sufficient time for the energy exchange among the target electrons. The most prominent two-electron process in collision with charged particles is that of double-ionization (*eg* Grujić, 1986) (double bound-free transition) near threshold, but this is of little importance for the problem at hand.

5.1.2 On-energy-shell transitions

Various semiclassical configurations exhibit different degree of stability and may pass from one to the other. In fact only three (Langmuir oscillatory, braiding torus and frozen-planet) configurations possess absolute stability and in principle other configurations might be just transient states on the route to these (and possibly other yet undiscovered) ones. For example, the so-called symmetrical collinear model (Ezra *et al.* 1991) turns out unstable and passes into more stable so-called asymmetric stretch configuration. Generally, higher degree of symmetry implies greater instability and *vice versa*.

There is one important class of on-the-shell transitions, when one of electrons is de-excited and the other moves into continuum - so-called *autoionization* or *Auger transition* (see figure 3). It is this process against which the stability is defined, since many double excited states are, as a rule, imbedded into continuum. It is also this process which competes most frequently with radiative transition probabilities.

We note finally that so far two-electron atoms have been considered. If the system has more than two electrons, calculations are not so simple. However, as both classical (Dohčević *et al.* 1991) and quantum mechanical (Dourneuf and Watanabe, 1990) calculations for He^- indicate the residual electronic core need not influence noticeably the life-time of the doubly-excited state. This may be explained by presumably incoherent perturbations by the inner part of the atom.

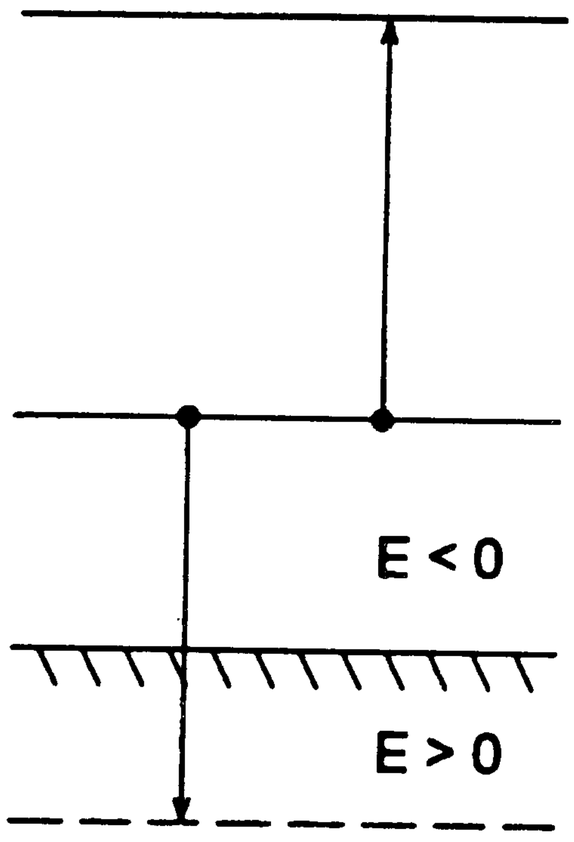

Figure 3. Bound-free two-electron transition, schematic.

5.2 Selection rules

Since a complete and accurate theory of two-electron states is still lacking, rules which govern non-radiative transitions are still of phenomenological nature, obtained by observing strengths of particular class of transitions, or by analyzing matrix elements in the corresponding sets of equations, within molecular-orbitals approach (*eg* Rost and Briggs, 1991). They are called, therefore, ***propensity rules.*** Non-radiative transitions are induced by non-diagonal matrix elements and they imply

$$\Delta t = 0, \quad (\textit{no parity change}) \tag{15}$$

$$\Delta m = 0 \quad (\textit{radial coupling}), \tag{16}$$

$$\Delta m = \pm 1 \quad (\textit{rotational coupling}) \tag{17}$$

One can show that a more restrictive sequence of selection rules (*via* radial coupling) holds

$$\Delta n_\mu = -2, \qquad \Delta n_\lambda = 0, \qquad \Delta m = 0 \tag{18}$$

The general rule is that $N - manifold$ decays preferentially to nearest lower one $(N-1)$, which then may dissociate into electron plus atom (or ion), if possible energetically. All these sequences end up in the state $n_\mu = 0, 1$. The latter may decay further by (weaker) rotational transition

$$\Delta n_\mu = 1, \qquad \Delta n_\lambda = 0, \qquad \Delta m = -1 \tag{19}$$

within a UA n-manifold. These transitions imply, in their turn, a change in A quantum number ($\Delta A = -1$), exchanging state with node at the saddle with that with an antinode and *vice versa*. The weakest of all transitions are those between different saddle sequences

$$\Delta n_\lambda \neq 0, \tag{20}$$

Finally, the same approximation of the electron-centre-of-mass motion around the saddle leads to the following ***dipole selection rule***

$$\Delta \nu_2 = 0, \tag{21}$$

5.3 Radiative transitions

As mentioned above, stability against non-radiative transitions decreases with N (non-radiative decay rates may decrease exponentially) and at some critical value N_{cr} radiative-decay rate equals that of non-radiative one. In the case of frozen-planet model, for example, $N_{cr} \approx 20$, what implies a lifetime of approximately $0.5 \times 10^{-6} s$ (Richter *et al.* 1992). For higher states ($N > N_{cr}$) doubly-excited states decay predominantly *via* radiative transitions.

Within the *dipole approximation* the radiative rates are given by (see *eg* (Chen and Lin, 1993))

$$W_R(10^{12}/s) = 2.75 \times 10^4 \frac{\alpha^3 \omega_{if}^3}{2L_i + 1} \cdot \sum_{M_i M_f} | < \Psi_f | \mathbf{r}_1 + \mathbf{r}_2 | \Psi_i > |^2, \tag{22}$$

where ω_{if} is the transition energy. This is to be compared with the corresponding Auger rates

$$W_A(10^{12}/s) = 2.58 \times 10^5 | < \Psi_f^c | r_{12} | \Psi_i > |^2, \tag{23}$$

where Ψ_f^c *is a product of a bound and a continuum single electron* wavefunctions. From (22) one sees that large photon energies are favoured. On the other hand, Auger transitions favour final continuum states with small-energy free electron ejected (Chen and Lin, 1993).

5.3.1 Selection rules

The new quantum numbers and the underlying interelectron correlations determine new classification schemes for sorting out spectroscopical data. Within Herrick-Lin designation ${}_N(K,T)_n^{A\ \ 2S+1}L^\pi$ one can distinguish for different spectral series. No general selection rule have been established up to now, but calculations for Ar^{16+} helium-like ion, in the initial state ${}_6(5,0)_6^{+\ \ 1}S^e$, have indicated a number of propensity rules presumably of general validity. As shown by Gou *et al* each of these series pertain to its own selection rules (Gou *et al.* 1991). Here we shall quote some of the relevant results from the same reference.

Table 5. Numerical results for transitions from ${}_6(5,0)_6^+ \quad {}^1S^e$ state of Ar^{16+} (Gou *et al.* 1991).

N	Auger energy (Ry)	Auger rate ($10^{12}/s$)	Branching ratio (×100)	Radiative rate ($10^{12}/s$)	Branching ratio (×100)
1	306.397	0.25	0.7	4.11	11.0
2	63.397	2.79	7.5	.84	2.2
3	18.397	7.94	21.2	.29	0.8
4	2.647	20.96	56.0	0.14	0.4
5				0.07	0.2
Total		31.94	85.4	5.45	14.6

Rydberg series : All quantum numbers, except n, are kept fixed. Unlike ordinary (single-electron) Rydberg series, this sequence comprises states with more than one single electron configuration. In Table 7 we quote calculated results for radiative and Auger transitions.

As can be seen from Table 5 radiative transitions tend to populate lower levels, contrary to the Auger ones. The most important consequence is that autoionization produces predominantly small-energy Auger electrons, leaving the bound electron in an highly excited state. This feature of non-radiative double transitions will be of some importance to our problem of line broadening, as we shall see below. Propensity rule for both radiative and Auger transitions is $\Delta\nu = 0$. This is explained by the charge distributions in the initial and final states, which concentrate around collinear configurations ($\theta_{12} = \pi$).

Rotor series : Here quantum numbers L, S, π are varied. Each successive member differs for one unit of angular momentum and opposite parity (π) and multiplicity (S). Higher L imply higher rotational excitations (up to a L_{max}). Propensity rule $\Delta\nu = 0$ holds here, too. In addition, one has $\Delta S = 0, \Delta L = 1, 0, -1$, whereas the parity π changes as well. Similar to the singly excited states transitions $L \rightarrow L - 1$ dominate those with $L \rightarrow L + 1$. As for the autoionization widths, they remain approximately constant along the series, except at the ends (Chen and Lin, 1989).

Vibrational series : All quantum numbers are constant, except the vibrational quantum number $\nu = N - K - 1$, which assigns to wavefunctions different nodal structure along the mutual angle θ_{12}. The partial autoionization and radiative rates for six states within $6l6l^1S^e$ manifold have been examined by Gou *et al* (Gou *et al.* 1991). The radiative rate appears quite small and Auger yield is almost unity (except for the last member of the series). Propensity rule $\Delta\nu = 0$ applies here too. Even if $\Delta\nu = 0$ is not dominant, the corresponding oscillator strength is larger than for large $\Delta\nu$ transitions.

Double-Rydberg series : This belongs to sequence with $N = n$ and the following fixed quantum numbers : ν, T, L, S, π. Contrary to the last series, this has a fixed number of θ_{12} nodes. This series appears as the most straightforward generalization of the standard Rydberg series, being of strictly intrashell nature. It is interesting for this series that the autoionization widths do not decrease smoothly as N is increased, as calculations for ${}_N(N-1,0)_N^+$ manifold show (see figures 6,7 in Gou *et al.* (1991)). A sharp rise in the Auger width may occur when the Auger electron energy is small. This implies that autoionization process can be an efficient mechanism for providing small-energy Auger electrons.

All these calculations have been restricted to the intrashell states ($A = 1$). They provide the general selection rule

$$\Delta\nu = 0, \tag{24}$$

In the case of systems with $A = -1, 0$ (as for small-Z systems) where the fluorescence yield need not be negligible, the above results may not hold. Calculations for the photoabsorption of H^- and He (Sadeghpour, 1991) predict for the vibrational quantum number (bending mode for the three-body rotor) ${}_n\nu^A = (n-K-T-1)/2$, which counts the nodes in θ_{12} mode.

$$\Delta\nu^+ = 0, \tag{25}$$

for the photo-excitations. These calculations suggest that both high-lying doubly-excited state excitations and autoionizing (and autodetaching) processes favour selection rule (25). Finally, we mention here recent calculations by Nicolaides and coworkers of partial widths for He^- two-electron ionization ladder resonances (Chrysos *et al.* 1992) and partial and total autoionization widths for a number of three-electron atom states (Themelis and Nicolaides, 1994). Calculations show that the coupling with neighbouring states may affect strongly partial widths, but usually has negligible effect on the total widths.

6. LINE BROADENING

The problem of spectral line broadening may be divided into two main tasks : (i) finding emission rates of a radiating atom in the presence of an external perturber and (b) calculations of the line shapes in the environments which impose a stochastic-like sort of perturbation (statistical part) (Regemorter, 1972). We first expound some relevant results from the first (stationary) part of the problem.

6.1 Stationary perturbations

For the sake of systematic it is convenient to subdivide the single-perturber problem into two classes: (i) properties of unperturbed emitter regarding emission probabilities and (ii) rate of emission under an external perturbation (Griem, 1974).

6.1.1 Stationary properties

These concern primarily evaluations of various matrix elements, necessary for obtaining transition rates (22). Since for highly-excited states both semiclassical and quantum theory apply, we shall present typical calculations within each of approaches.

Semiclassical theory

Following Bohr's correspondence principle classical frequences for transitions between levels with high principal quantum numbers (and not too small angular momenta of the Keplerian orbits) are good approximations to the quantal ones, provided that (Percival, 1972)

$$\Delta n << n \tag{26}$$

For single-electron transitions one obtains for the oscillator strength (see *eg* Percival, 1972)

$$f(n',n) = \frac{8}{3}\frac{J_s(s)J_s'(s)}{s^2}\frac{n'^2}{n+n'}, \qquad s = n-n', \quad n' < n \tag{27}$$

where J and J' are Bessel function and its derivative. Generally, one has for any classical function F_c (see *eg* Percival, 1972)

$$< n'|F|n > \approx F_s^c \tag{28}$$

under the same conditions as in (28), where F_s^c are (classical) Fourier components of F_c matrix elements. For two-electron transitions we quote recent calculations of semiclassical hyperspherical Coulomb matrix elements by Mahecha (see Mahecha Gomez, 1994 for details) in Table 6

Table 6. Semiclassical (SC) and quantum mechanical (QM) Coulomb matrix elements $< n_{rc}l_1l_20M|C(\alpha,\theta_{12})|n'_{rc}l_1l_20M >$ for the hyperspherical Hamiltonian for helium S states, together with the corresponding quantum mechanical results (Mahecha Gomez, 1994).

(l_1l_2)	(0,0)		(1,1)		(2,2)		(3,3)	
(n'_{rc},n'_{rc})	SC	QM	SC	QM	SC	QM	SC	QM
(0,0)	12.09	11.118	9.85	9.39	9.18	8.79	8.82	8.47
(0,2)	4.63	3.69	2.12	2.12	1.46	1.60	1.16	1.35
...	...	...	...	...	...	...	...	...
(4,6)	9.58	9.23	5.43	5.51	3.91	4.04	3.09	3.21
(6,6)	21.46	21.19	16.61	16.55	14.64	14.60	13.48	13.46

Table 7. Polarizabilities (α) and hyperpolarizabilities (γ) for helium doubly-excited states and for some other cases.

state	$\alpha(au)$	$\gamma(au)$	$\alpha(au)$	$\gamma(au)$
	$M_L = 0$		$M_L = \pm 1$	
$He[2s^2(^1S^e)]$ [a]	75.5	2.9×10^5		
$He[2s2p(^3P^o)]$ [a]	41.2	1.2×10^5	130.8	-7.0×10^5
$H^-[1s^2(^1S^e)]$	206	8.02×10^7 [b]		
$He[1s^2(^1S^e)]$	1.383 [g,h]	43.104 [c]		
	1.389 [d]	42.78 [d]		
$He[1s2s(^1S^e)]$	803.			
$He[1s2s(^3S^e)]$	316.			
$Li[1s^22s(^2S^e)]$	164.9 [d]	3.7×10^4 [d]		
	164.8 [c]	-2.8×10^3 [c]		
	164.1 [e]	3×10^3 [e]		
$Be[1s^22s^2(^1S^e)]$ [f]	37.30	3.15×10^4		

(a) Ref. Nicolaides and Themelis (1993a); (b) Ref. Pipin and Bishop, 1992a; (c) Ref. Marulis and Thakkar, 1989; (d) Ref. Nicolaides and Themelis 1993b; (e) Ref. Pipin and Bishop, 1992b; (f) Ref. Thakkar, 1989; (g) Ref. Buckingham and Hibbard, 1968; (h) Ref. Bishop and Lam, 1988.

Quantum mechanical calculations

As for the quantum mechanical theory we mention here elaborate calculations by Wintgen and Delande (1993) of the double photo-ionization of $^1P^o$ states of helium. Making use of the so-called *complex scaling techniques* they were we able to evaluate accurate values for a number of decay widths, transition strengths and shape parameters for a number of Rydberg series converging to different inner electron N-manifolds. These results appear to be in good agreement with recent high quality experimental findings by Domke *et al.* (1991).

6.1.2 Atom in external fields

As the principal mechanism for line broadening in doubly excited atoms is the Stark effect, behaviour of these metastable systems in an external electric field is of primary interest. If a neutral S-state atom is a subject to an external, axially symmetric electric field F_z with gradient F_{ZZ}, its change in energy is given by Buckingham (1967)

$$\Delta E = -\alpha_d F_z^2/2 - \alpha_q F_{zz}^2/8 - \beta F_z^2 F_{zz}/4 - \gamma F_z^4/24 - \ldots \quad (29)$$

where $\alpha_d, \alpha_q, \beta$ and γ are dipole, quadrupole polarizabilities and dipole-dipole-quadrupole and dipole hyperpolarizabilities, respectively (Urban, 1987). One of the main goals in studying atomic systems perturbed by electric fields is, of course, evaluation of the dipole polarizability $\alpha(\alpha_d)$, which plays a prominent role in determining *polarization potential* for charged-particle - atom interaction

$$V_{pol} = a(r)\frac{\alpha}{2r^4}, \quad a(r) \to -1, r \to \infty \quad (30)$$

where $a(r)$ may be a complicated function for small and medium values of the charged-perturber - atom distance r. For neutral emitters this is the dominant long-range interaction (Christophorou and Illenberger, 1993) and at small collisional energies influences crucially the classical trajectory of the perturber (*e.g.* Dimitrijević and Grujić, 1978; Dimitrijević and Grujić, 1979; Dimitrijević, 1989).

In a recent paper Nicolaides and Themelis (1993a) carried out calculations of the polarizabilities and hyperpolarizabilities of two of the lowest helium doubly excited states, by computing complex eigenvalues for suitably chosen non-Hermitian matrices. In Table 7 we give the values for the polarizabilities from Nicolaides and Themelis (1993a), together with results for other helium states and for other atoms, for the sake of comparison.

Before we comment results in Table 7, few remarks seem in order. First, singly-excited states have larger polarizabilities than the corresponding ground state, because of weaker influence of the nucleus on active electron. Second, α may be negative for some

excited states (repulsive asymptotic interaction). And third, triplet states have, as a rule, larger α than the corresponding triplet states, because of stronger interelectron repulsion in the latter. For the same reason, doubly-excited states have smaller α than the corresponding singly-excited atoms, since here electron correlations join the Pauli-effect electron repulsion, resisting, for example, collinear configurations bending in the external field.

As can be seen from Table 7 dipole polarizablities as calculated by various authors (using different methods) agree reasonable well, as different from the hyperpolarizabilities, which are much larger and may differ even by the sign. In the same paper Nicolaides and Themelis (1993a) calculated half-widths of the doubly-excited states of helium. They found an configuration mixing effect of *decreasing* widths as the electric field rises. Since their wavefunctions were constructed from many configurations ($MCHF$ method), neighbouring states with smaller widths influenced the state at hand in decreasing its width. Thus an interesting effect arises : an external perturber, by mixing in other states of the atom, stabilizes it, by prolonging its life-time. This phenomenon may be of importance for the line broadening mechanism.

We note here that the dipole polarizability of doubly excited atoms can be obtained within (semi)-classical picture. In the recent paper Mahecha (Mahecha Gomez, 1989) carried out calculations for helium, within two classical configurations, *electron top* and *electron rotor*. For weak fields one obtains analytically

$$\alpha(rotor) = 0.30(L_z/Z)^6 a_0^3, \qquad (31)$$

$$\alpha(top) = 1.22(L_z/Z)^6 a_0^3, \qquad (32)$$

where L_z is the projection of the angular momentum onto the field-direction, and Z is the nucleus charge.

In some situation higher order effects should be included, like the quadrupole polarization, quadrupole and higher moments of the emitter *etc.* To our knowledge no calculations of these quantities have been reported up to now.

6.2 Line broadening mechanisms

Compared with the single-excited atoms, double-excited systems provide much richer variety of de-excitation routs. Non-radiative transitions compete and/or interfere with radiative ones and a number of new de-excitational channels appear. We shall distinguish two main kinds of radiative de-excitations: (a) single-electron radiation and (b) two-electron emission.

6.2.1 Single-electron radiation

As noted before, autoionizing states tend to desintegrate by two-step processes : (i) first an electron is ejected into continuum (Auger electron) and (ii) the other electron de-excites and radiate a photon. As emphasized above many Auger processes tend to populate low-energy continuum electron states, leaving the other electron in highly excited state. The latter has small energy width and high probability to radiate in passing to a lower state. Since the Auger electron moves away slowly, it is also very probable that the radiative transition occurs while the outgoing electron is sufficiently close to influence the emitter radiation process. Thus, Auger electron serves as an external perturber. This mechanism may be called *autobroadening*, what should not be confused for *selfbroadening*, of course, which arises when both perturber and emitter are of the same kind (*e.g.* Leo *et al.* 1995). It belongs to the class of the so-called *post-collision interaction* processes, which have been studied extensively in the last fifteen years (see *e.g.* Gouw, 1995).

Unlike foreign perturber Stark broadening, which involves perturbers of various energies and impact parameters (Barat and Roncin, 1992; Seaton, 1995), Auger electrons have all the same energies and zero impact parameters. The only statistical variable here appears the moment of radiative transition, as measured from the instant of the Auger (first step) transition. Of course, for many emitters in a plasma, one has a number of various Auger discrete levels, closely spaced, what can make calculations for realistic systems far from easy. So far, to our best knowledge, no calculations along these line have been carried out.

6.2.2 Two-electron radiation

This should be a standard generalization from the common single-electron line broadening. Two electron transitions, governed in the free-emission case by (22) rate of emission, are perturbed by plasma electrons (ions). As some calculations indicate, these external perturbations may be (un)favourable to fluoresence yield, by mixing initial doubly-excited states (a sort of radiation-(de)focusing effect). It is interesting to note here that the semiclassical theory predicts both regular and irregular (chaotic) double-excited states. The external perturbation may serve as a (de)stabilizer of the latter. For example, it has been found that an external stochastic noise may stabilize otherwise unstable (chaotic) classical system. Coupled with the time-dependence of line-broadening elementary processes, this phenomenon may have profound effects on the Stark broadening of plasma lines. Further, perturbation by ions introduces additional collisional channels, like the single and double electron capture, as well as the so-called *capture into continuum*, what would further complicate line broadening investigations.

6.2.3 Mixed cases

Of course, with number of excited electrons rising, more complex processes may come to light. An interesting example has been reported recently by Ehresmann *et al.* (1993), in what they call three-electron radiative transitions. They investigated theoretically a single-photon decay of doubly-vacancy-one-electron states in KrI. They followed experimental observations by Ehresmann *et al.* (1992), when by the photon-induced fluoresence spectroscopy method a new group of weak lines was observed, in the region $(540 - 375\mathring{A})$, after resonant excitation of $KrI3d^{-1}np$ $(n = 5, 6)$ states. They proposed the model which describe an initial state with two holes and three excited electrons and final states with double-hole and the third electron in a new discrete state plus a photon, *i.e.* the transition

$$i_1 i_2 n \rightarrow f_1 f_2 n' + \gamma, \qquad (33)$$

Third electron participates very weakly in the transition (it may retain its energy, but not the angular momentum) and may be considered to be a spectator, promoting only the selection rule for double-electron transition. The latter may be of a true multistep nature, with a number of intermediate states involved. Actual computations have been carried out for $KrII$ transitions $4s^{-2}np \rightarrow 4p^{-2}n'l' + \gamma$. The predicted line intensities are far below the presentday experimental sensitivity, but the model offers interesting possibilities in interpreting highly correlative few-electron transitions. In particular, it suggests discrete electron and photon spectra, together with the continuous one.

This process appears interesting for us, for it exemplifies a perturbed two-electron transitions, just as one would expect this to takes place in a plasma medium. In fact, third electron considered in the above example need not necessarily be from the emitter, but plasma surrounding may provide a perturber to exchange the angular momentum with atoms and allow a previously forbidden radiative transitions.

Triply excited states have been already observed in lithium by photoabsorption experiment (Kiernan *et al.* 1994), and a number of theoretical papers on the subject (mainly on He^-) have been published (*e.g.* Bylicki, 1992; Nicolaides *et al.* 1993) see also (Grujić and Simonović, to be published) for a more complete reference list on triply and quadruply excited states).

7. DISCUSSION AND CONCLUDING REMARKS

Studies of doubly excited atoms is a fast developing research area in atomic physics and much efforts to elucidate their properties have been made both theoretically and experimentally. So far these studies have been restricted mainly to free systems and the task of exammininings interactions with external environments remains yet to be fulfilled. Two principal questions must be answered within these studies.

First is a specification of exact conditions under which doubly excited states participate in collective processes to the extent that their contribution to the macroscopic system properties must be considered. For not too large principal quantum numbers of either of excited electrons, one may expect that doubly excited atoms play a noticeable role even in laboratory plasmas. As for the highly excited atoms, similar to the situation with ordinary Rydberg systems, one expects that the playground for these very large atoms must be searched in an astrophysical environment.

Since under more common conditions it is reasonably to expect doubly excited atoms to constitute an "excited minority" within the plasma systems, and consequently provide weak emission lines, much progress in improving experimental sensitivity in detecting and analyzing the corresponding radiation spectra is expected. Situation with the absorption processes is much more favourable, of course, since an experimental set up can be arranged to enhance radically double-excitation processes. It is this transition from controlled collisional experiments to statistical studies in a random motion media, like plasmas, which makes the study of the multiply excited systems radiation a difficult task indeed.

The second question to be answered in future research is the implementation of the tools, already developed in studying excited atoms under external perturbations, to the new situation when another class of final (and intermediate) states exists - those of autoionization (Auger processes). On the more formal ground, incorporation of the semiclassical models in calculating induced radiative and non-radiative transitions would be desirable. This particularly concerns highly excited states, where the (semi)classical theory appears very successful. Semi-classical configurations applied to the targets in collisional studies have already proved useful. In a sense, highly excited systems and charged environment constitute a semiclassical realm *par excellence* and here one would expect a semiclassical theory to be effective indeed. When we speak of the semiclassical theory here we mean a *proper semiclassical theory*, which would involve both emitter and perturbers, not just the latter (as the case with the standard semiclassical approximation is (Regemorter, 1972)).

Finally, we note that it seem to us that the whole field of multiply excited systems has matured enough for making the next natural step from free to (plasma) perturbed conditions. If this lecture contributes to encouraging researchers in the line broadening field to undertake studies of the emission from multiply excited states, we would feel our effort to draw this topic to their attention to be worth while.

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ДВОСТРУКО ПОБУЂЕНИ АТОМИ И ШИРЕЊЕ СПЕКТРАЛНИХ ЛИНИЈА

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Прегледни чланак

Атомски системи са двоструко побуђеним електронима предмет су интензивног изучавања последњих петнајест година. Ова аутојонизациона стања могу бити дугоживући метастабилни атоми, са неким особинама специфичним за системе где међуелектронске корелације играју круцијалну улогу и модел независних честица постаје неадекватан већ у нултој апроксимацији.

Овде ћемо изложити основне механизме за образовање оваквих стања, навести битне црте двоструко побуђених атома и најчешће начине за њихов распад. Посебна пажња биће посвећена методама за добијање енергијског спектра, како квантно-механичким тако и семикласичним прилазима за рачунање положаја и ширина спектралних линија. Биће наведене могуће моде радиационих прелаза и дискутовани неки механисми ширења спектралних линија. Најзад, скицираћемо неке правце развоја теорије ширења оваквих линија.