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VISUAL BINARY ORBIT DETERMINATION. ONE POSSIBLE APPROACH.

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SUMMARY: We present a method for determining orbital elements of visual binaries from observed relative positions. This method is especially suitable for deriving preliminary orbits, and we called it the method of falsified observation (FO) (Ćatović, Olević, 1992.). Here we give extensive discussion of theoretical and practical merits of the proposed method.

1. INTRODUCTION

Visual binary orbit calculation is a very old problem. Many methods for solving it were suggested, but all of them are "good" in some particular but not in all situations. Let us summarise briefly what are the major obstacles for constructing methods which would be effective generally.

The *major difficulty* in solving the orbital motion of the visual binary is drawing, or placing, the apparent orbit. Once the apparent orbit is placed, then geometrical elements $(a, e, i, \Omega, \omega)$ of the orbit are known. On the other hand, since the time is not involved in placing the apparent orbit, dynamical elements (P, τ) are not determined yet. Therefore, the *second difficulty* consists in associating the epochs of the observed positions to their calculated counterparts on the apparent orbit. It is essential problem as well and we will discuss it in more details in the following chapters.

In our opinion, every try to formulate a new approach should take into account the following requirements:

Requirments following from the theory:

i) Since we are interested in physical binaries with elliptical orbits, the first requirement comes from *geometry of the orbit* - i.e. the orbit must be an ellipse.
ii) Since orbital motion of physical binaries is Keplerian motion the second requirement comes from *dynamics of the orbit* - i.e. the second Kepler law must be valid for chosen orbit.

Requirements following from the tools used:

(By this we mean which analytical, or other tools we used to define an apparent orbit.)

i) Heuristic determination of the apparent orbit, usually based on the calculators experience, is often used in practice. It requires carefull examination of input data, or choosing three observations and areal constant for methods of Thiele - Innes type.

ii) Method of least squares (MLS) could be used, as objective mathematical tool to approach the problem, but it requires long enough and curved enough observed arcs in order to provide the ellipticity of the solution.

Since computational equipment and techniques are very well developed and since equations of condition (geometrical and dynamical) are known from theory in the case of visual binaries, MLS seems the best tool for solving the problem of visual binary orbit determination. On the other hand, however, since observed arcs are usually small MLS can not guarantee an elliptical solution. If we take into account the inaccuracy of the input data too, then even if ellipticity is not questioned, we can expect there more than one orbit which would be sufficiently accurate. It would be interesting, therefore, to search for the method by which we can "check" more than one orbit for the given observations.

The most used methods in astronomical practice are described in the classical texbooks on visual (and spectroscopic) double star astronomy such as Aitken (1935), Coutou (1975), Heintz (1978). Eichhorn and Xu (1990) suggested an improved algorithm for solving an orbital motion of visual binaries based on equations of condition which include geometrical and dynamical conditions simultaneously. These equations are highly nonlinear and transcendental but the authors used MLS algorithm Eichhorn (1985) which overcomes this inconvenience. However, it should be mentioned that this method needs initial solution provided by some other method. MLS algorithm converges if that initial approximation is sufficiently accuarte.

Dommanget (1978, 1981) gave a general method (for all eccentricities) for determination of visual binary orbit based on three fundamental points and areal constant i.e. based on method of Thiele - Innes - van den Bos. At the Royal Observatory, Belgium, this method is widely used for computing the orbital elements of visual binaries (Nys, 1983.) This group of methods is based on heuristic determination of three (among all other) fundamental observations as well as areal constant *a priori*.

2. EQUATIONS OF CONDITION

Theoretical basis for the method we suggest here is linear MLS. By this procedure, which is divided into two parts, one can exemine large number of elliptical orbits associated with one pair. In the first, geometrical part, elliptical orbit is determined by MLS (we denote thereafter this part of the procedure GMLS). Equations of condition are equations of conical section in the rectangular coordinates, where all of the elements of the orbit appear as linear parameters.

$$z_1x_i^2 + z_2y_i^2 + z_3y_ix_i + z_4x_i + z_5y_i + 1 = 0; \quad i = 1, ..., N. \tag{2.1}$$

where N is the number of observations; $x_i = \varrho_i \cos\theta_i$, $y_i = \varrho_i \sin\theta_i$ are the observed apparent positions and $z_1, ..., z_5$ are unknown coefficients - geometrical elements of the orbit. Having in mind the obstacle to direct use of MLS, mentioned in the previous chapter, we introduce here the idea of falsified observation (FO) (Ćatović and Olević, 1992). FO is simply an arbitrary point in xy plane, chosen by the calculator in order to force MLS to give elliptical solution. The orbit should pass through (or very close to) FO and fit the regular observations by satisfying the least squares principle. Therefore the choice of the position of FO is the crucial point in this procedure.

In the second, dynamical part (this part of the procedure hereafter denoted DMLS), the mean motion and the moment of passing through periastron are determined by employing linear MLS again; this time equations of condition are connections between mean anomalies and the epochs of observations

$$nt_i - \beta = M_i; \quad i = 1, ..., N, \tag{2.2}$$

where $\beta = n\tau$; $M_i = E_i - e_i \sin E_i$. Mean anomalies M_i are calculated with the help of orbital elements obtained by GMLS as if they lay on the calculated elliptical orbit (which is not true). Therefore we will call these mean anomalies $M_i = M_i(x_i, y_i, a, e, i, \Omega, \omega)$, observed mean anomalies. However, since the original observations (observed apparent places) are not placed exactly on the apparent orbit (calculated by GMLS), neither will their projections on the orbital plane, be placed on the true orbit exactly. In this procedure we use observed mean anomalies for calculating n, τ.

2.1 Further potentialities of equations of condition

In this subsection we will disscus further possibilities for the equations of condition, having in mind the idea of introducing FO. If we include FO as regular observation (but with much larger weight) than equations of condition for GMLS would be the equations (2.1). The number of degrees of freedom, or number of unknown coefficients, in this case is five.

But if we demand that elliptical orbit pass through FO exactly than the number of unknown coefficients can be reduced by one. Namely, then equation

$$z_1x_f^2 + z_2y_f^2 + z_3y_fx_f + z_4x_f + z_5y_f + 1 = 0, \tag{2.3}$$

where x_f, y_f are coordinates of FO, holds. Then we can calculate one of the coefficients $z_1, ..., z_5$ as function of coordinates of FO and other coefficients. Obviously, now there are five different possibilities for equations of condition, and here, we will write these equations.

$$z_1 \cdot 0 + z_2\left(y_i^2 - \frac{y_f^2}{x_f^2}x_i^2\right) + z_3\left(x_i y_i - \frac{y_f}{x_f}x_i^2\right) +$$
$$z_4\left(x_i - \frac{x_i^2}{x_f}\right) + z_5\left(y_i - \frac{y_f}{x_f^2}x_i^2\right) = -\left(1 - \frac{x_i^2}{x_f^2}\right),$$
$$z_1\left(x_i^2 - \frac{x_f^2}{y_f^2}y_i^2\right) + z_2 \cdot 0 + z_3\left(x_i y_i - \frac{x_f}{y_f}y_i^2\right) +$$
$$z_4\left(x_i - \frac{x_f}{y_f^2}y_i^2\right) + z_5\left(y_i - \frac{y_i^2}{y_f}\right) = -\left(1 - \frac{y_i^2}{y_f^2}\right),$$
$$z_1\left(x_i^2 - \frac{x_f}{y_f}x_i y_i\right) + z_2\left(y_i^2 - \frac{y_f}{x_f}x_i y_i\right) + z_3 \cdot 0 +$$
$$z_4\left(y_i - \frac{x_i y_i}{y_f}\right) + z_5\left(y_i - \frac{x_i y_i}{x_f}\right) = -\left(1 - \frac{x_i y_i}{x_f y_f}\right),$$
$$z_1(x_i^2 - x_i x_f) + z_2\left(y_i^2 - \frac{y_f^2}{x_f}x_i\right) + z_3(x_i y_i - x_i y_f)$$
$$+ z_4 \cdot 0 + z_5\left(y_i - \frac{y_f}{x_f}x_i\right) = -\left(1 - \frac{x_i}{x_f}\right),$$
$$z_1\left(x_i^2 - \frac{x_f^2}{y_f}y_i\right) + z_2(y_i^2 - y_i y_f) + z_3(x_i y_i - y_i x_f)$$
$$+ z_4\left(x_i - \frac{x_f}{y_f}y_i\right) + z_5 \cdot 0 = -\left(1 - \frac{y_i}{y_f}\right). \tag{2.4}$$

Index i counts observations in all five equations (i.e. $i = 1, \ldots, N$). In case of equations (2.4) coeffitients $z_1, \ldots z_5$ are calculated from:

$$z_1 = -z_2\frac{y_f^2}{x_f^2} - z_3\frac{y_f}{x_f} - z_4\frac{1}{x_f} - z_5\frac{y_f}{x_f^2} - \frac{1}{x_f^2},$$
$$z_2 = -z_1\frac{x_f^2}{y_f^2} - z_3\frac{x_f}{y_f} - z_4\frac{x_f}{y_f^2} - z_5\frac{1}{y_f} - \frac{1}{y_f^2},$$
$$z_3 = -z_1\frac{x_f}{y_f} - z_2\frac{y_f}{x_f} - z_4\frac{1}{y_f} - z_5\frac{1}{x_f} - \frac{1}{x_f y_f},$$
$$z_4 = -z_1 x_f - z_2\frac{y_f^2}{x_f} - z_3 y_f - z_5\frac{y_f}{x_f} - \frac{1}{x_f},$$
$$z_5 = -z_1\frac{x_f^2}{y_f} - z_2 y_f - z_3 x_f - z_4\frac{x_f}{y_f} - \frac{1}{y_f}, \tag{2.5}$$

respectively.

As for DMLS there are some possibilities for different approach as well. This time the type of equations of condition remains, but the problem of association of the anomaly to the epoch could be solved in different way. For instance, if we associate apparent observed position (x, y) to its counterpart (x', y') on the apparent orbit which is placed on the intersection of the apparent orbit and line drawn from the primary A, to the apparent place B (see Fig. 1). Then we can project that point - intersection on the true orbit (with elements obtained by GMLS) in order to obtain mean anomaly $M' = M'(x', y', a, e, i, \Omega, \omega)$ and associate it to the epoch of observation.

Another possibility is to associate the apparent observed position (x, y) to its counterpart $(\tilde{x}, \tilde{y})$ on the apparent orbit which is placed at the minimal distance of the point B to the apparent orbit (see Fig. 1). Then the corrensponding mean anomaly, associated to the epoch of observation t, will be $\tilde{M} = \tilde{M}(\tilde{x}, \tilde{y}, a, e, i, \Omega, \omega)$

Although we did not compare results of these different aproaches yet, we beleive that it could be interesting for further investigation. The problem of association of the anomaly to the epoch did not receive attenion yet (to our knowledge) although it is an essential problem.

Finaly, let us note that DMLS part of the procedure is implicitly dependent on the solution by GMLS. It gives the best solution (in the sense of the least squares) for n and τ if true places (apparent places projected by parameters obtained by GMLS) and the corresponding epochs are known.

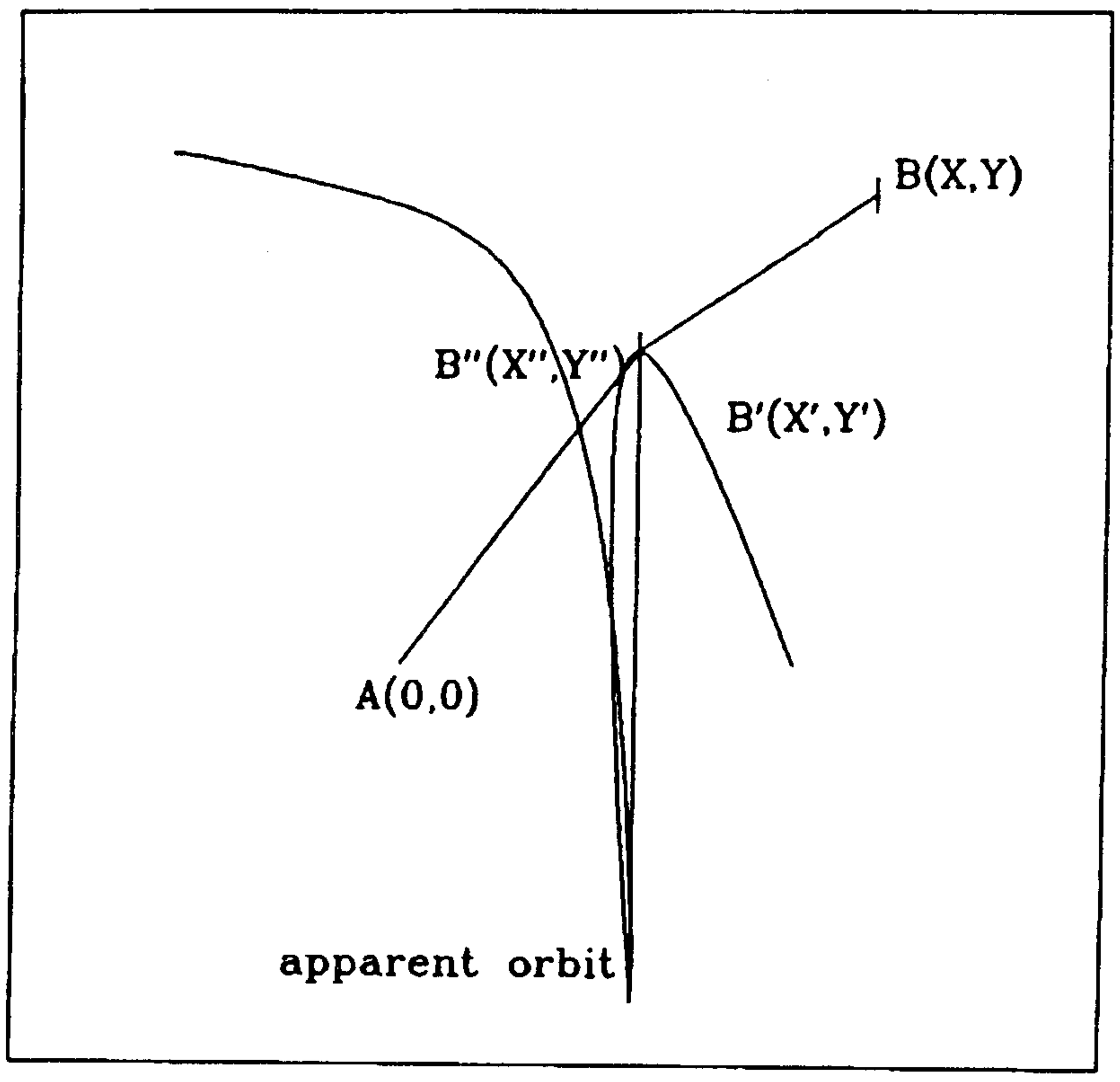

Figure 1.

3. EXAMPLE

On the basis of principles discussed in the previous chapter we developed a FORTRAN (MS FORTRAN 5.1running on PC) routine for calculating orbital elements of visual binary. Equations of condition, in this case, are equations (2.1), which means that FO is treated as regular observation with great weight (equal to the sum of weightes of all other observations). Equations of condition for DMLS are equations (2.2) and M_i-s are observed mean anomalies discussed in the previous chapter.

First, let us demonstrate how the programme works with simulated observations. Table 1a contains simulated apparent places and $(O-C)$ correnspondig to two calculated orbits. Figure 2 shows these orbits plotted on the tangent plane. Table 1b shows input orbital elements, and two sets of output orbital elements, corrensponding to two FO-s. The first one is placed on the apparent orbit exactly, and input elements are returned. The second is shifted from the apparent orbit. Table 1b contains also areal constants calculated for three arbitrary chosen areas. This subroutine (calculation of areas of elliptical sectors) in the programme is very usefull in comparing orbits. It could reveal which orbit fulfills law of areas better.

Table 1a. Observations (test example)

n	time	ϱ_{2000}	θ_{2000}	$(O-C)_{\varrho_1}$	$(O-C)_{\theta_1}$	$(O-C)_{\varrho_2}$	$(O-C)_{\theta_2}$
01	1995.50	0.721	108.714	.0000	-.0008	.1112	17.5311
02	2003.05	0.855	150.238	.0000	.0014	.0976	1.0039
03	2010.60	0.997	179.540	.0000	-.0009	.0177	-2.9277
04	2018.15	1.098	202.386	.0000	.0004	-.0218	-2.5725
05	2025.70	1.172	221.897	.0000	-.0010	-.0299	-1.5185
06	2033.25	1.236	239.263	.0000	.0000	-.0216	-0.6465
07	2040.80	1.297	254.962	.0000	.0009	-.0075	-0.1679
08	2048.35	1.355	269.271	.0000	-.0002	.0054	-0.0429
09	2055.90	1.407	282.454	.0000	.0005	.0130	-0.1295
10	2063.45	1.445	294.807	.0000	-.0004	.0134	-0.2799
11	2070.99	1.462	306.679	.0000	.0002	.0065	-0.3634
12	2078.54	1.451	318.495	.0000	-.0007	-.0063	-0.2837
13	2086.09	1.400	330.819	.0000	.0000	-.0216	0.0464
14	2093.64	1.302	344.529	.0000	.0008	-.0333	0.7021
15	2101.19	1.149	1.223	.0000	-.0004	-.0294	1.6889
16	2108.74	0.946	24.242	.0000	.0010	.0104	2.3517
17	2116.29	0.754	60.009	.0000	-.0017	.0888	-2.5926

Table 1b. Orbital elements and areal constant

input elements		output elements$_1$		output elements$_2$	
$a = 1''.213$	$\Omega = 168^\circ.49$	$a = 1''.213$	$\Omega = 168^\circ.49$	$a = 1''.249$	$\Omega = 178^\circ.68$
$e = 0.329$	$i = 31^\circ.23$	$e = 0.329$	$i = 31^\circ.23$	$e = 0.392$	$i = 37^\circ.31$
$P = 128.34$ yr.	$\omega = 269^\circ.48$	$P = 128.34$ yr.	$\omega = 269^\circ.48$	$P = 124.22$ yr.	$\omega = 286^\circ.47$
$\tau = 1995.5$		$\tau = 1995.50$		$\tau = 1997.49$	
		$\varrho_{f_1} = 0.7209668$	$\theta_{f_1} = 108.713513$	$\varrho_{f_2} = 0.60$	$\theta_{f_2} = 110.0$
		$AC_{1,3} = 0.0290831$		$AC_{1,3} = 0.0234916$	
		$AC_{15,17} = 0.0290831$		$AC_{15,17} = 0.0268590$	
		$AC_{9,11} = 0.0290831$		$AC_{9,11} = 0.0285662$	

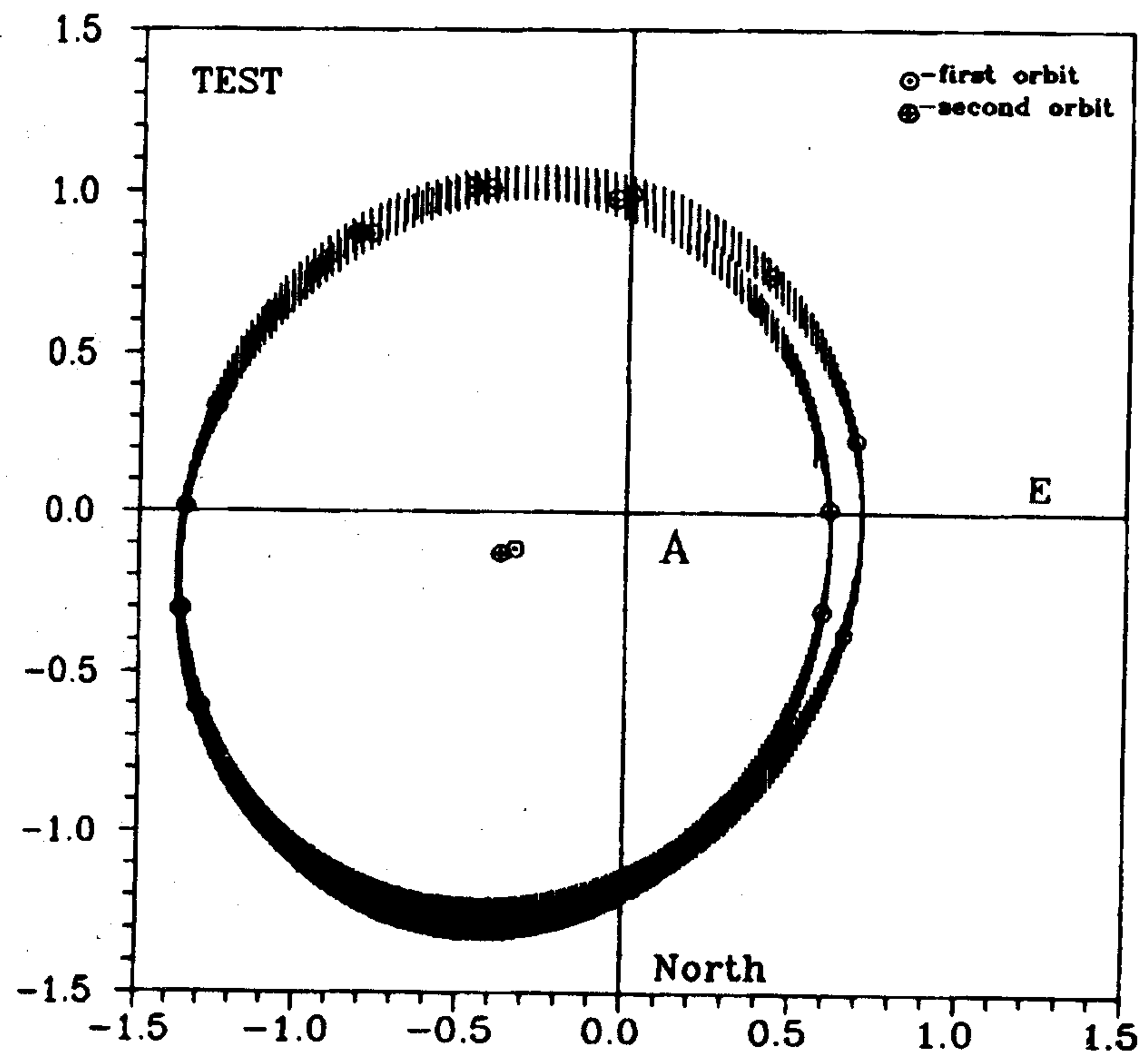

Figure 2.

4. CONCLUSION

From theoretical point of view, the introduction of FO opens two very important questions:

a) can we determine the area of FO's which provide elliptical solutions?
b) if answer to question a) is positive, are we able to select those FO's, among all, which fullfill dynamical condition?

Choice of FO could be conditioned in some way but the problem is that every try to do so perturbes the linearity of equations of condition (2.1) or (2.2) and (2.4), and therefore complicates the procedure. On the other hand, nowadays it must not be treated as a problem since extensive computations could be done easily. In any case, it seems to us that this problem still deserves some attention of the researches. Furthermore, different types of equations of condition (2.1), (2.4), should be studied more from both practical and theoretical points of view.

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REFERENCES

Aitken, R. G.: 1935, *The Binary Stars*, Dover Publ. Inc., New York.

Ćatović, Z. and Olević, D.: 1992, *in Astron. Soc. of the Pacific, Conference Series*, **Vol 32**, Atlanta.

Couteau, P.: 1978, *L'Observation Des Étoiles Doubles Visuelles*, Flammarion, Paris.

Dommanget, J.: 1978, *Astron. Astorphys.* **68**, 315.

Dommanget, J.: 1981, *Astron. Astorphys.* **94**, 45.

Eichhorn, R.: 1985, *Astrophys. and Space Science*, **110**, 119.

Eichhorn, R. and Xu, X.: 1990, *Astrophys. J.* **358**, 575.

Heintz, W.D.: 1978, *Double Stars*, Dordecht, D. Reidel Publ. Co.

Nys, O.: 1983, Programmes for computing visual binary orbits from three fundamental points and the apparent areal constant. *Comm. Obs. Roy. de Belgique, Serie B* N° 128.

Xu, X.: 1992, Doctoral dissertation, University of Florida, Gainesville.

ОДРЕЂИВАЊЕ ОРБИТА ВИЗУАЛНО ДВОЈНИХ ЗВЕЗДА. ЈЕДАН МОГУЋИ ПРИЛАЗ.

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Оригинални научни рад

У раду је представљен један поступак одређивања путањских елемената визуално двојних звезда методом најмањих квадрата користећи "лажно" посматрање (falsified observation – FO). Метод је погодан за одређивање прелиминарне орбите. У раду се дискутују теоријске и практичне предности предложеног поступка.