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Review paper

NEWTONIAN AND COULOMBIC SYSTEMS

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SUMMARY: A comparative study of few- body systems with gravitational and electromagnetic interaction is given, with emphasis on similarities and distinctions of a number of relevant characteristic features. Recent advances in investigations of multiply excited atomic and planetary celestial systems are presented and a number of some interesting analogies are discussed.

1. INTRODUCTION

The formal similarity between the gravitational and electrostatic interactions has been noticed for a long time, for both types of forces are expressed by the same mathematical function

$$F_{ij} = \lambda_{ij} r_{ij}/r_{ij}^3, \; r_{ij} = r_i - r_j \qquad (1)$$

with r_{ij} as a distance between two interacting bodies. The difference appears in the *coupling constants* λ_{ij}, which in each particular case reads:

$$\lambda_{ij} = q_i q_j \quad (Coulombic\ case) \qquad (2)$$

$$\lambda_{ij} = \gamma m_i m_j \quad (Newtonian\ case) \qquad (3)$$

where q_i, q_j are the point charges and γ, m_i, m_j are gravitational constant and body masses, respectively (all in appropriate units). Strictly speaking, these laws refer to the asymptotic interactions between real physical systems, more precisely for the mutual separations much larger than the dimensions of the interacting subsystems. In the realm of atoms and molecules, whose structure and interactions are governed by the Coulombic force, nonelectrostastic interactions appear when charged subsystems are brought into close contact, which can alter the simple power law (1) radically, e. g. change from an attractive to repulsive and *vice versa*. It is this interplay between Coulombic and chemical forces which makes the material world stable, to a degree depending on the temperature (i. e. the phase state) of the system. But even in the case of the so-called elementary particles, like electron and proton, the electromagnetic interaction deviates considerably at very close particle separations (Drell, 1969) specific effects, like the so-called zero-point fluctuations, vacuum polarization etc, make the interaction potential for $r_{ij} = 0$ finite. These effects, which are responsible for a number of subtleties within the atomic physics are dealt within the quantum electrodynamics (QED), which is based on the notion of exchange of virtual particles rather than on the concept of line of force, the latter being the (classical) basis of the interaction-at-distance concept of force.

Similarly, Newton's law is valid only, for not too strong gravitational force, otherwise one should turn to the Einsteinian theory of gravitation, i. e. to the General (Theory of) Relativity (GR). The latter situation arises when bodies with extremally large densities closely interact, so that r_{ij} can be very small, as the case with the hypothetical black holes is. We shall deal, however, with extended celestial bodies and shall likewise ignore other relativistic effects, like the small deviation of the simple power law (1) due to the mass variabillity etc.

So, we shall confine ourselves to the most simple physical situations when all interacting bodies (particles) may be considered as pointlike, structureless objects, so that the force law is given by relation (1).

Because of the formal similarity, it is tempting to ascribe to both types of interactions the same underlying physical source of the for-ce. Indeed, among the so-called fundamental interactions: gravitational, electromagnetic, weak and strong, it was two first which were first considered for an unification. As it is well known, this attempt failed. Moreover, it is clear now that the gravitational force is fundamentally different from the rest and if the so- called Grand Unification (GU) of all fundamental interactions will ever be accomplished, the gravitational field will be the last to join the other three. On the other hand, the electro-magnetic interaction has been successfully coalesced with the weak to make the so-called *electroweak* interaction, and there is a good prospect that this will be joined by the strong interactions to make the so-called Unified theory, the present *Standard model* of elementary particles. Another essential difference between the gravitational and other interactions is the difficulty to formulate a quantum theory of gravitational field. While it has been one of the greatest successes of the modern physical theory to establish an extremally accurate quantum theory of the electromagnetic field (QED) and construct a number of very prospective theories of weak and strong interactions, notably the so- called quantum chromodynamics (QCD), despite enormous efforts by theoreticians the gravitational field has defied all attempts to merge two greatest intelectual achievements of our century: GR and Quantum mechanics (QM). Moreover, it is not yet clear whether the gravitation is a proper force at all, or only an aspect of the space-time manyfold, as asserted by GR. In the light of this puzzle the problem arises whether one should first set up a more general QM which incorporates *ab initio* the structure of the space-time manyfold, or try to quantize the gravitational field by the standard QM (see, e. g., Linden, 1990). The latter approach has resulted in suggesting a hypothetical gravitational quantum, *graviton*, in analogy with the (electromagnetic) photon, but all searches for the gravitational waves, which should be subsequently quantized, have turned out to be futile up to now. Nevertheless it is customary now to regard both electromagnetic and gravitational interactions *via* exchanges of virtual bosons – photons and gravitons, respectively. Being of infinite range, both forces are conceived to realize by massless bosons, with the difference that the photon has an intristic angular momentum (spin) equal one, whereas the graviton has spin 2. Apart from the strong interaction, which is mediated by the masless hypothetical quanta called *gluons*, the infinite range of electromagnetic and gravitational forces makes it possible to formulate suitable potential functions of both interactions and apply the standard mechanisms of QM and classical potential theories in treating corresponding physical systems.

If the gravitational interaction appears somewhat apart from the other three regarding its strength and vague status as a force, electromagnetic field has, in distinction from the gravitational and strong forces, the unique property that the mediating quanta (photons) are not coupled with the interacting particles (charges), being electrically neutral. On the other hand, being coupled with gravitating masses, they can be distracted by the latter from its straight -line trajectories, the fact which yields the procedure to ascribe to the physical space a dynamic feature, as done within GR, which thus geometrisassed gravitational field.

If this interaction between the electromagnetic and gravitational field furnishes the mean to determine the global space-time structure, the combined action of the universal gravitation and Coulombic forces (including those derived from it, like various kinds of chemical forces, magnetic interaction, etc) is responsible for the large-scale structure of the matter, like the celestial bodies, galaxies, etc. Generally speaking, this cooperative influence on the shape of the inert matter is determined by the balance noticed as one goes from the bulk matter to the microworld level: the smaller the material entity is, the more prominent the Coulombic force becomes, and the smaller influence of the gravitational mass on its motion is. As a consequence, at the planetary level the electromagnetic force appears completely insignificant, while at the atomic level it is only inertial mass of the particles which matters. This fact is a consequence, or rather a sign of the enormous difference in numerical values of the electromagnetic and gravitational coupling constants, which differ by approximately 40 degrees of order. In the situations when the Coulombic force is eliminated at the very microscopic level, by mutual neutralization of elementary particles, like electron and proton, the equlibrium is destroyed and the selfgravitating matter ascquires properties, like those of the hypothetical neutron stars, which go beyond our present experience, including those of the black holes. We shall confine ourselves, however, to the more common matter conditions, where the identity of the ordinary elementary particles is ensured.

2. GRAVITATIONAL AND ELECTROSTATIC INTERACTIONS

Despite the enormous difference in strengths, both interactions have a number of common properties, which explain partly why they play so dominant roles in shaping the matter at macro- and micro-levels, respectively. We enumerate some of these features of the inverse square law force:

(i) It is the only power-law (apart from the harmonic-force law, see e g. Blitzer, 1988) which enables one to treat the spherical bodies as the point particles in the most relevant physical situations.

(ii) Only the inverse-square law provides the most pleasant property of the spherical distribution of mass and charges that the interior of this spherical layer appears force-free (that is, the potential is constant).

(iii) Because of the particular (Euclidian) structure of the real three- dimensional physical space, the (power) square law allows for the important physical quantity – the line of force – to be defined for the point sources. Moreover, it can be shown that the real space must have exactly three dimensions, if the potential theory is to work at all. This can be the best appreciated by considering the two-dimensional spherical (noneuclidian) space, as often done for expounding some ideas from GR.

(iv) Beside the linear (harmonic oscillator) force, with $n = 1$, it is only $n = -2$ power law: $F = \lambda r^n$ which ensures the closed orbits for the motion around, the force centre. This important result stems from the so- called *Bertrand's theorem*, which asserts that only Young and Newton forces support stable (against external perturbations) periodic orbits. As pointed out by a number of authors (e. g. Goldstein, 1981) only from this theorem and from the observational evidence concerning the remarkable stability of the planetary motion Newton could have inferred his law of the universal gravitation, without resorting to any calculations (discarding, of course, Young force, on general physical grounds). This common feature of the gravitational and electromagnetic force has been clue for formulating the so-called ***planetary model*** of atomic systems due to Rutherford and Bohr, from which the *Quantum mechanics* has arisen as the final step.

(v) These forces appear as interactions with the longest range in nature. In this respect, they are the only ones which exclude the asymptotes to the zero- energy binary system configurations, within the classical dynamics, or correspondingly exclude the plane-wave state function in the quantum-mechanical formulation, for any energy (e. g. Newton, 1966).

(vi) It is this particular mathematical structure of the Coulombic force which furnish a number of fundamental relations between the classical and quantum mechanical descriptions of the Coulombic systems, which are known as the ***corresponding (principle) identities (e. g.*** Norcliffe, 1975). The latter make the analogy between the celestial and atomic physics less accidental, than it is generally believed, as we shall see later on.

Another important common property (though, possibly, not quite unique among physical interactions) is their pair-wise additivity. If the system consist of N bodies, with binary interaction V_{ij}, then the potential function can be written as:

$$V(\{r_i\}) = \Sigma_{i<j} V_{ij} \tag{4}$$

where $\{r_i\}$ stands for all positions of the constituents. Hence, no many-body forces must be invoked in order to describe the behaviour of such many-body systems. This is by no means general property, as examplified in nuclear physics, for instance, molecular physics (Margenau and Kestner, 1975) etc.

Besides the *difference in strengths*, there are other fundamental disimilarities between the gravitational and electrostatic interactions. The most important is the dimensionality of corresponding 'spaces': there are two kinds of the electrical charge, whereas only one sort of gravitating mass is known in Nature. As a consequence two charged particles either attract or repel each other, whereas two neutral masses undergo attraction only. However, this is not always true. Better to say, it is as true as the statement that the light is the fastest signal possible, for there are situations when massive particles move faster than the electromagnetic waves, as in a dispersive medium. We shall elaborate this in a somewhat more detail, starting with gravitating bodies.

For the sake of simplicity, consider two identical spherical masses, emersed in an infinite, ideal (nonviscous) fluid, with density ρ_f (see Fig. 1). We distinguish two cases with regard to the particle density ρ_p:

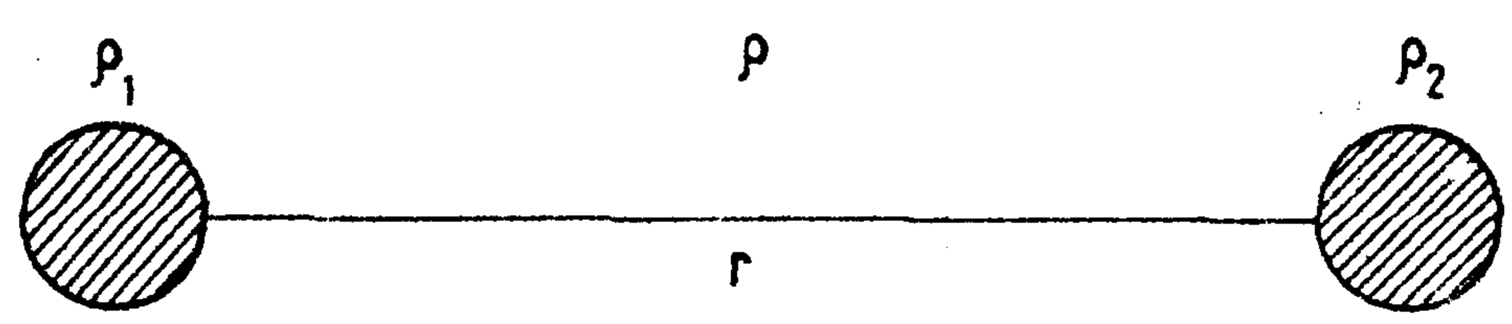

Fig. 1. Gravitating bodies in an ideal fluid.

(a) $\rho_\pi > \rho_f$. The force F_{12} will be the standard Newton's attraction: - $\gamma m_1 m_2/r_{12}^2$, but the fluid will exercise its influence too (see, e. g. Horak, 1984) providing a negative contribution to the primary attraction, since, for instance, the influence of m_2 on m_1 will be diminshed by the 'ghost' mass m_2' symmetrically placed, as shown in Fig. 1, and similarly for m_2. Obviously, in the limmit $\rho_p \rightarrow \rho_f$, no net interaction will occur.

(b) $\rho_p < \rho_f$. In this case the net interaction will be repulsive one, as can be easily inferred from Fig. 1, by the same reasoning (we note that the assertion in Horak, 1984, that the force will be again attractive is wrong, as can be deduced also by the 'centrifuge argument', quoted by the same author).

Horak (1984) also shows that in the case of different body masses and if $\rho_1 >_{\rho f}$, $\rho_2 <_{\rho f}$ (or *vice versa*) the net force will be repulsive also. If $\rho_p = 0$ one can construct a 'dynamics of bubbles' in a fluid, but we shall not dwell on it here.

In the similar manner, one can demonstrate that, for instance, two equal charges will attract each other, if emersed in a charged fluid of the greater charged density, etc.

These effects can bear some consequences on the measuring of gravitational constant γ in a medium, like air. Some large scale phenomena of the type just described may be of some importance to the cosmological model used currently, as described by Voraček (1985). In fact, one could construct a particular cosmological model based on the 'bubbles dynamics', but this would be out of scope of the present paper.

The principal consequence of the drastic difference in magnitudes of coupling constants is that the particles with Coulombic interaction undergo large accelerations and acquire also large velocities, as examplified by the atomic electron motion and as different from the macroscopic and celestial bodies. As a direct outcome there is a qualitative difference in treating processes and systems at the ordinary (and astronomical) and microscopic level: while, at normal conditions, one employs the (post)Newtonian theory for describing macroworld dynamics, in the realm of atoms and elementary particles one resorts to the quantum mechanical formalism, as examplified by Schrödinger equation. However, there are situations where the Newtonian dynamics appears inadequate, as mentioned above, i. e. when the gravitational force becomes unusually strong, as is the case with (mini) black holes, and one turns to the more adequate GR, with possible quantum effects included. On the other hand, atomic and subatomic systems can be brought into states where the (semi)classical dynamics applies, as the case with highly excited atoms is. It is these regions of weak forces which we intend to discuss and compare here: the region of (semi)classical dynamics for the Coulombic systems and of the (post) Newtonian approximation of the gravitational systems. As we shall see a number of significant analogies and striking similarities can be revealed for these apparently very distinct systems.

One important property of the power-law interactions is the homogeneity of the potential functions:

$$V(\{\theta r_{ij}\}) = \theta^k V(\{r_{ij}\}) \qquad (5)$$

where θ is a real number. Now, if the system motion obeys Newton's laws, then the following ***homothetic*** properties (the so-called scaling laws) hold

$$r_{ij} \rightarrow \theta \cdot r_{ij} \qquad (6a)$$

$$t \rightarrow \theta^{1+k/2} \qquad (6b)$$

$$E \rightarrow \theta^{-k} E \qquad (6c)$$

$$L_i \rightarrow \theta^k L_i \qquad (6d)$$

where E is the total energy, and L_i the angular momentum. In the case of the inverese-distance interaction systems (IDIS), $k = -1$, (6a) and (6b) provide the similarity transformations of the form:

$$r_{ij}(t) \rightarrow \theta r_{ij}(\theta^{3/2} t) \qquad (7)$$

In other words, if $r_{ij}(t)$ is the solution of Newton's (or any other equivalent) equation for a particular system, then the system obtained by reducing ($\theta > 1$) the energy will expand in space and the motion becomes appropriately slowed down, but the shape of all trajectories are unchanged (homothetic transformation). These scaling laws provide an effective way for deducing a number of system properties at various energies, once the classical problem is solved for a particular E. For instance, from (7) one easily obtains the third Kepler's law for the planetary motion. Similarly, (6d) determines all the semiclassical energies within the Bohr's *Old Quantum Theory*, once a particular (quantized) orbit is evaluated (numerically or otherwise).

Many physical systems have a homogeneous potential functions, of the type

$$V = \Sigma_{i<j} \lambda_{ij} / r_{ij}^k, k \neq 0 \qquad (8)$$

at least in the asymptotic regions, $r_{ij} \gg 1$ (in appropriate units), like the ansemble of neutral atoms ($k = 6$), charmonium systems ($k = -1$). For IDIS, relativistic terms of the order $0(r_{ij}^{k+1})$ spoil the system homogeneity. In many situations, however, these terms, which give rise to the relativistic energy splitting for the Coulombic systems and to the Laplace's vector precession of the planetary motion, may be ignored. This is particularly so in the positive energy regions, ($E > 0$), especially close to the fragmentation threshold ($0 \leq E << 1$), when the motion at large interparticle separations is relevant, as we shall see later on.

Accounting all the similarities and differences between the gravitational and electromagnetic interactions, we may expect these features to reflect into the properties and behaviour of the corresponding physical systems. We shall treat first the simplest of them – binary systems.

3. TWO-BODY SYSTEMS

Ignoring for the moment the influence of an eventual surrounding medium on the IDIS, as discussed above, the principal difference between Newtonian and Coulombic binary systems is that, in the absence of an 'antigravitation', the former are governed by the attractive and the latter by both attractive and repulsive forces. Since the repulsive interaction doesn't support bound states, we shall confine ourselves to those Coulombic systems with opposite charges and compare the properties of two-body systems at the negative energy, $E < 0$.

At the level of binary systems, we notice another remarkable common feature of the inverse-square-distance and linear forces: besides the highly specific $1/r^3$ force, these are the only power-law interactions which admit the classical solution *via* elementary (nonelliptic) functions. The trajectories of the motion of the equivalent mass around the effective fixed centre-of-force (at the centre-of-mass) are ellipses in both cases. For the harmonic force, $F = -kr$, the centre of ellipse coincides with equilibrim point, whereas the attractive centre-of-force for the inverse square law lies at one of ellipse foci. It is interesting that both $k = -2, 1$ cases in Eq. (8) can be formally treated as the harmonic oscillator problem, after a suitable transformation of coordinates, both within the classical (e. g. Broucke, 1980) and quantum mechanical (e. g. Chen and Kibler, 1985) formalisms. In the light of the fact that the harmonic force arises naturally within few-body IDIS, as we shall see later on, this doesn't seem an accidental feature.

Elementary mechanics shows that for the particles acting *via* central force, two-body problem can be reduced to reduced mass $\mu = m_1 m_2/(m_1 + m_2)$ (harmonic mean) moving around the centre-of-mass. In the orbit plane (which is fixed by the orientation of the angular momentum L) one has generally three integrals of motion: energy E, angular momentum L and the initial time moment t_0. Since it is the system with four degrees of freedom one expects another constant of motion. In IDIS one indeed finds the fourth constant (e. g. Park, 1979)

$$A = \lambda \vec{r}/r - 1/m(\vec{p} \times \vec{L}) \tag{9}$$

the vector which points from the focus towards the apoapsis (see Fig. 2). This vector has been explicitely introduced by Laplace (1798), but the result had been already contained in the work by J. Bernuli in 1710. Within the Quantum mechanics of the one-electron systems this vector is known as Runge-Lenz (sometimes Pauli) vector. The appearance of the fourth integral is a direct consequence of the so-called *accidental degeneracy*, independence of the energy on the angular momentum. An addition of a noninverse square force removes this degeneracy, and makes the orbit open and rotates A. The

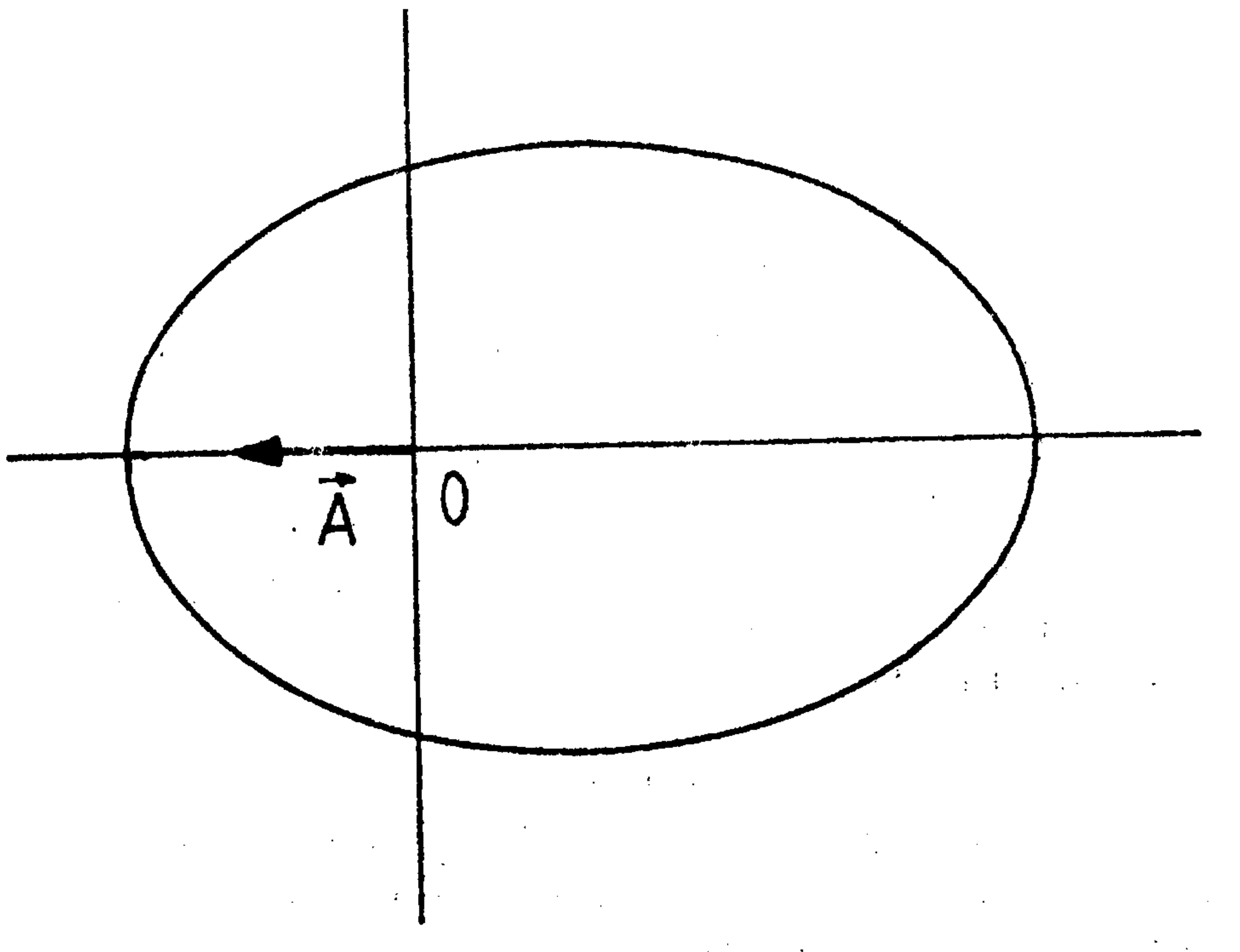

Fig. 2. Keplerian orbit with the Laplace vector **A**.

presence of constant A ensures an additional stability of the planetary motion and greatly simplifies quantum mechanical two-body problem (Pauli, 1926). Had not A existed, discovery of both *Old Quantum Theory* and modern *Quantum mechanics* would have been considerably delayed. Very few physical systems are endowed with all possible constants of motion (e. g. Kibler and Witnernitz, 1990) and one of major preoccupations of the few-body theoreticians is the search for these missing integrals.

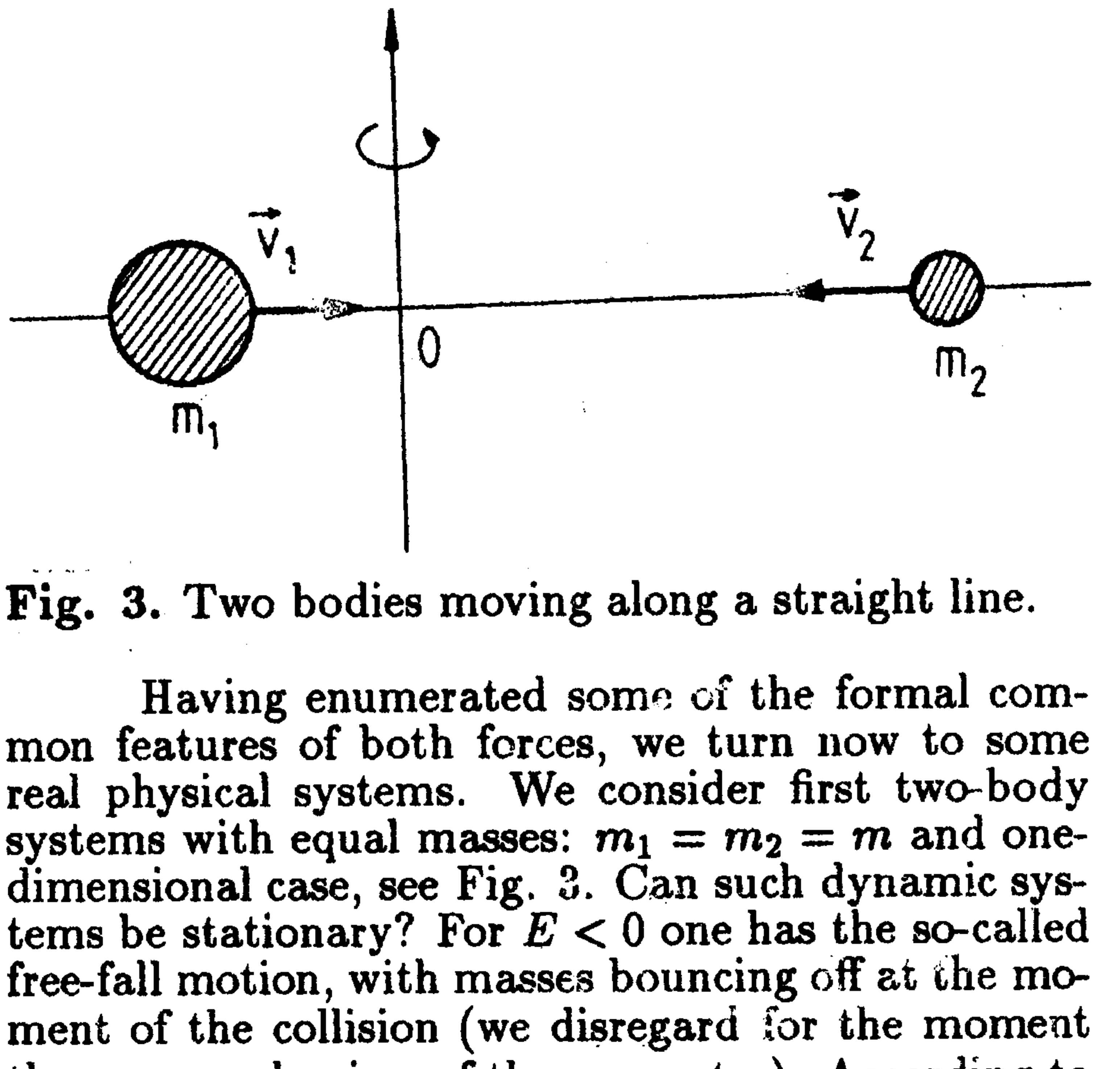

Fig. 3. Two bodies moving along a straight line.

Having enumerated some of the formal common features of both forces, we turn now to some real physical systems. We consider first two-body systems with equal masses: $m_1 = m_2 = m$ and one-dimensional case, see Fig. 3. Can such dynamic systems be stationary? For $E < 0$ one has the so-called free-fall motion, with masses bouncing off at the moment of the collision (we disregard for the moment the very mechanism of the encounter). According to GRT acceleration of a gravitational mass gives rise to an emission of gravitational waves. In this particular configuration the emission is forbidden since the conservation of the (linear) momentum excludes the dipole radiation. The only way to produce a gravitational wave here is by rotating masses around the axis of symmetry, as indicated in Fig. 3 (quadrupole emission). That the 'breathing motion' does not allow the loss of energy is the fact of utmost importance in majority of cosmological models.

On the other hand, dipole radiation of charged systems, due to the time variation of the dipole moment, is the most common mechanism for producing an electromagnetic field, at the atomic level. On the macroscopic scale, one has dipole antennas, which can be represented schematically by the masses in Fig. 3, endowed by mutually opposite charges q_1, q_2.

The binary linear systems display some important differences between Newtonian and Coulombic systems, but the real systems to compare are atomic and planetary ones and we shall devote particular attention to these problems. Because of the inter-electron strong interaction the planetary model of atoms (Percival, 1977) requires much more disperse configurations than the corresponding celestial systems. Another possibility would be an atomic model with many electrons revolving around the (heavy) nucleus on the same circular orbit, as proposed originally by Bohr (1913) in his early papers on the atomic structure, but we shall discuss this and some other configurations in the next chapters.

For the single-electron circular orbit Bohr discovered a remarkable *quantum rule* for the possible energy levels (we use the so-called *atomic units:* $e = m = \hbar = 1$)

$$E_n = -1/2n^2, \quad n = 1, 2, 3, ... \qquad (10)$$

with circle radius given by

$$r_n = n^2 a_0 \qquad (11)$$

where r_0 is the radius of the so-called first Bohr orbit (ground state level): $a_0 = 0.529\ 10^{-8}\ cm$. In the further elaboration of the planetary model (Bohr-Sommerfeld theory) elliptic trajectories are introduced, bringing the model closer to our Solar system configuration. It is interesting that Bohr excluded zero-angular momentum configurations (free-fall case) from the very beginning, on the ground that the electron and nucleus would collide otherwise. In his theory the angular momentum quantum assumed the values

$$l = 1, 2, 3, ..., n \qquad (12)$$

for the fixed *principal quantum number* n, whereas the latter exact Quantum mechanics due to Heiseberg and Schrödinger yielded

$$l = 0, 1, 2, ..., n - 1 \qquad (13)$$

that is, the only excluded from the theory were those proposed originally by Bohr, circular paths with $l = n$. Although it is based on a quinte a different conceptual basis than the Old Quantum Theory, without reference to the notion of (classical) trajectory, it is instructive to see the way the quantum mechanical description explains the "decent" behaviour of the two-body system, within the Schrödinger picture, for otherwise singular Coulombic potential problem, with the pole at $r = 0$. For the 'free-fall configuration' in the hydrogen atom ground state ($l = 0$), one can easily evaluate the associated de Broglie wavelengths at $r \simeq 0$ and it turns out that, e. g. for the hydrogen ground state ($E = -1/2\, au$), $\lambda = \hbar/p \simeq 8\ 10^{-3}\ a_0$. So it exceedes the (classical) electron (proton) radius by two orders of magnitudes. Hence, the electron "doesn't see" the nucleus as a particle. Consequently, there is little point in comparing Coulombic (microscopic) systems with the gravitational (macroscopic) ones for the (almost) linear configurations, for the underlying physical pictures of the constituents behaviour appears quite different, as described by radically different theories. There are, however, situations where the atomic systems are well described by the (semi)classical theory and we shall devote our attention to these conditions.

If the angular momentum quantum number l is large, what may occur for large n, as seen from (13), the classical trajectory picture becomes more and more adequate and in the limit $l(n) \rightarrow \infty$ (Bohr's *correspondence principle region*) the classical and quantum mechanical results coincide. Since, according to (11) dimensions of the atom become rapidly enormous for large n, highly excited hydrogen-like atoms are difficult to produce in the laboratory. On the other hand, one does observe the radio emission lines corresponding to $n + 1 \rightarrow n$, $n \sim 100$ transitions (Percival and Richards, 1975), from the interstellar regions, in particular from the Solar corona, which implies that the atoms as large as 1μ can be found in the extremally rarefied space. However, usually one still has $l << n$, so that these planetary atoms should be compared with comets, rather than with planets of our Solar system. Nevertheless, the fact that Bohr's theory provides correct energy levels for the one-electron atoms, makes from the one hand, Coulombic interaction "more classical" than the other types of interactions and adds another property to the analogy between the Coulombic and Newtonian systems, on the other hand.

Being far from the two-body system our Solar system would be, from a formal point of view, a poor counterpart of the one-electron atom. However, because of the smallness of planetary masses, compared with the Solar one (in full analogy with the ratio between the electron and nuclear masses) and the smallness of the gravitational constant G, the Solar system can be considered as consisting from binary subsystems, with the Sun as a common member. Unlike the many-electron atoms, mutual interactions between the orbiting bodies may be regarded as small perturbations and the planetary motion as quasiindependent (we shall consider some interesting examples later on). What makes sense to compare is the overall structurally fixed configuration of the Solar planetary system with the hydrogen-like atom in various excited states. One notices first that the highly excited atom has normally large eccentricity orbit, whereas the planets as a rule are endowed with very small eccentricity trajectories. These properties come from the very mechanisms which the motion is produced by: in the atomic case, it is the ***angular momentum transfer rule:*** $\Delta l = \pm 1$ which forces excitations of not too large l states, whereas the almost circular orbits of majority of Solar components testify about the formation of planets out of a rotating primordial matter, as asserted by all realistic cosmological models.

Now, what's about the stability problem? Atoms appear stable *via* the quantization rules, which define well separated stationary orbits of the electrons. There is no similar theory that the could ensure stationarity of macroscopic systems like the Solar one (but see later). The later appears long lived more due to favourable initial conditions, which together with Kolmogorov-Arnold-Moser (KAM) theorem, ensures its existence within the cosmic time scale. In fact, if these initial conditions are not met, the strongly interacting bodies would distroy each other, as a number of the Solar constituents have already done. Two, the question arises: do the initial conditions plus these "correcting factors" bring about any regularity of the planetary structure, of the sort of (11) for instance? Many attempts have been made since antique times, to find out correlations in planetary motions. One of the earliest trials was due to Platon, who in his famous dialogue *Thimayos* divided the whole Cosmos into two 'worlds': *sub-lunar* and *astral* ones. Ascribing to the former spheres with radii (with the Earth in the centre): $2 \simeq \sqrt[3]{10}$ (*aether*), $5 \simeq \sqrt[3]{100}$ (*air*), $10 \simeq \sqrt[3]{1000}$ (*fire*), he asigned the following distances from the centre of Cosmos – Earth (1), Jupiter ($8 = 1 + 2 + 5$), Saturn ($13 = 1 + 2 + 5 + 5$), and stars ($18 = 10 + 8$). Since Platon, this sort of speculative "playing with numbers" have never seased and a number of rules (regularities) have been proposed. The most famous rule is the well known Bode's "law", which asserts that the mean distances of planets from the Sun can be given by the relation (in appropriate units)

$$a_n = 0.4 + 0.3 \times 2^n, \quad n = 0, 1, 2, \ldots \qquad (14)$$

Now, apart from the constant 0.4, (14) looks like an 'inverted quantum condition' (11), *ie* one has an exponential rather than the power "law". Recently, an attempt has been made to set up a corresponding 'quantum rule', analogies to (11) for the planetary radii, inspired by an ancient idea of Philolaos (Tomić, unpublished), in the form (in astronomical units)

$$a_n = n^2/215, \quad n = 1, 2, \ldots \qquad (15)$$

Of course, without an underlying thery, all these relations remain in the realm of speculations, but an analysis of the mechanism of creation of a Solar-like system might reveal preference to the regularities like (15), just as the present theories explain otherwise curious fact that all planets lie in a common plane.

The principal difference between, say, Bohr-Sommerfeld quantization rule, which appears as universal law, being expressed by universal constants of Nature: $\hbar$, e, m and relations type (14) or (15), is that the latter are only locally (and approximately) valid, *i. e.* they are expressed in local units. In other words, if there were celestial (cosmological) unit analogies to the abovementioned atomic-scale universal units, one could meditate on a sort of ***cosmic quantum mechanics***. Strange as it sounds, such attempts have been made and here we shall expound briefly some interesting results.

Following an asserted evidence for the so-called ***cosmic redshift quantization*** (*e. g.* Tift and Cocke, 1984) a number of theoreticians have been making attempts to establish a kind of ***cosmic quantum mechanics*** (*e. g.* DerSarkissian, 1985), by constructing a ***gravitational Planck constant***, an analogy of the ordinary (atomic scale) Planck constant $\hbar$. Thus a proposition is offered (DerSarkissian, 1984)

$$\hbar \simeq (1 + \sqrt{3})^2 m_g (\Delta v)^2 / H_0 \qquad (16)$$

where m_g is a typical cosmic mass, H_0 is Hubble constant and Δv is the velocity increment for adjacent red-shift states, which is considered constant in the so-called *Tift interval rule* and is assumed to acquire value of 72 km/sec, or a fraction thereof. This yields a numerical value of $\hbar \simeq 7\ 10^{74}$ $ergsec$, for $m_g = 10^{44}$ g, $H_0 = 50$ $km/secMpc$ and $\Delta v = 12$ km/sec. A search has been made for "Bohr's orbits" in double galaxies (DerSarkissian, 1986) with an indecisive result.

Yong-Zhen and Zu-Gan (1985) proposed a set of actions proper to gravitating systems

$$\hbar^{(s)} = (\hbar c^5 / 2\pi \hbar_0^2)^{1/6} (c^5 / 2\pi G H_0^2) \qquad (17)$$

where the ***scale parameter*** s acquires values 1, 2, 3 for galaxies, stars and larger asteroids, respectively. Their formula (which reduces to the ordinary Planck constant $\hbar$ for $s = 6$) however, has been criticised by DerSarkissian (1986) on the grounds that it doesn't ensures the conservation of the angular momentum as the cosmic time evolves. In order to remedy this unpleasant feature of the many-level universal constant Kaminisi and Arai (1987) have proposed an alternative expression

$$\hbar^{(s)} = [\hbar (H/2GcM_U^3)^{1/2}]^{s/6} (2GcM_U^3)^{1/2} \qquad (18)$$

which remains constant and reduces for $s = 6$ to well-known value of Planck's action, whereas for $s = 0$ (18) should refer to the very Cosmos, with mass M_U.

What is ***rationale*** for these highly speculative "astro-physical" models, including the very idea of "Cosmic Quantum Mechanics"? One of motivations, though implicite one, is the current interest in the so-called *cosmic strings* (*e. g.*, Tassie, 1986; see also Brandenberg and Turok, 1986). The idea is that celestial objects have evolved through a series of (hiearchical) breaking of rotating cosmic strings, the latter being a sort of topological defects of the underlying space-time manyfold (*e. g.* Kibble, 1987). This sort of forming cosmic objects ensures the relation $J = KM^2$, where M is the mass of the object and K a universal constant, which acquires a numerical value $\approx 10^{-15}\, g^{-1} cm^2 s^{-1}$ (comparable with the corresponding constant in the particle physics string theory). The breakdown of these gigantic "quantum objects" presumably gives rise to the creation of stars, galaxies and supergalaxies (but see Sivaram, 1987), thus bringing the cosmic-scale processes down to the atomic scale models (where Coulombic force dominates, instead of the gravitational one).

Parallel with these "observational-theoretical conjectures" there is a search going on for inferring from the astronomical evidence an underlying *fractal structure* of the Universe (Einasto *at al*, 1988, Saar, 1988; but see Martinez and Jones, 1990). This idea has been pushed to its extreme by Oldershaw (1989), who has proposed the so-called self-similar cosmological model, which correlates physical parameters from the atomic up to galactic- scale systems. It should be mentioned, however, that all these hierarchical structure inferences are of statistical-observational nature and unless a firm theoretical ground is established, they are deemed to remain interesting speculations only. One way towards an unifying picture would be a kind of unifying string theory (both the particle physics and cosmic ones) but this would go beyond the scope of the present article. We turn now to the more serious problem – the three-body systems.

4. THREE-BODY SYSTEMS

The full three-body problem has not been solved neither within the classical nor quantum mechanics. Though in the latter case there are now a number of methods, like the *Fadeev equations*, for example, which solve the problem in principle, the long-range nature of the inverse square law force makes all these approaches still far from being truly operative.

Within the classical dynamics three-body problem has been attacked first in the celestial mechanics, notably by Lagrange, who has formulated the problem rigorously and found a few particular solvable cases. In the realm of atoms, the problem is first met in attempting to evaluate the energy spectrum of the two-electron atoms, as it was done for the hydrogen like atoms – the famous *helium problem*. In connection with the dichotomy – classical *vs* quantum mechanical pictures, the problem has two aspects: (a) the calculation of the three-body motion with arbitrary initial conditions (the general solution): (b) the choise of initial conditions which ensure a reasonable stability of the system. As we mentioned above, the task (a) has not yet been achieved in either fields analytically for the gravitational and electromagnetic forces, though with the aid of modern computers numerical solutions are easily found. As for the point (b) the conceptual difference between the classical and quantum theories has separated the very problem posed: in the former one seeks appropriate initial values, whereas the quantum mechanical description of the atomic states requires an imposition of the boundary condition for the (stationary) states. Thus the seemingly unsurmontable problem of explaining the identity of all atomic systems with identical constituents has been solved by eliminating it as such.

Nevertheless, if one leaves aside ground states of the atoms, which are outside the realm of classical picture and turns to those states where the notion of classical trajectory gains meaning, though only approximately, we can find a lot of common features of the three-body gravitational and electromagnetic systems, as we shall see for a number of cases now.

Lagrange's cases

These refer to the configurations which can be regarded as quasi two-body problems. We shall distinguish three classes of configurations: linear, planar and three-dimensional.

(i) *Collinear case.* Let three masses and charges $(m_i,\, q_i)$, $i = 1,2,3$, be situated on a common straight line, as shown in Fig. 4, which we take to be oriented in the three-dimensional space with a unit vector n. If the coordinates in the centre-of-mass system are x_i, $i = 1,2,3$, one has

$$\Sigma m_i x_i = 0 \tag{19}$$

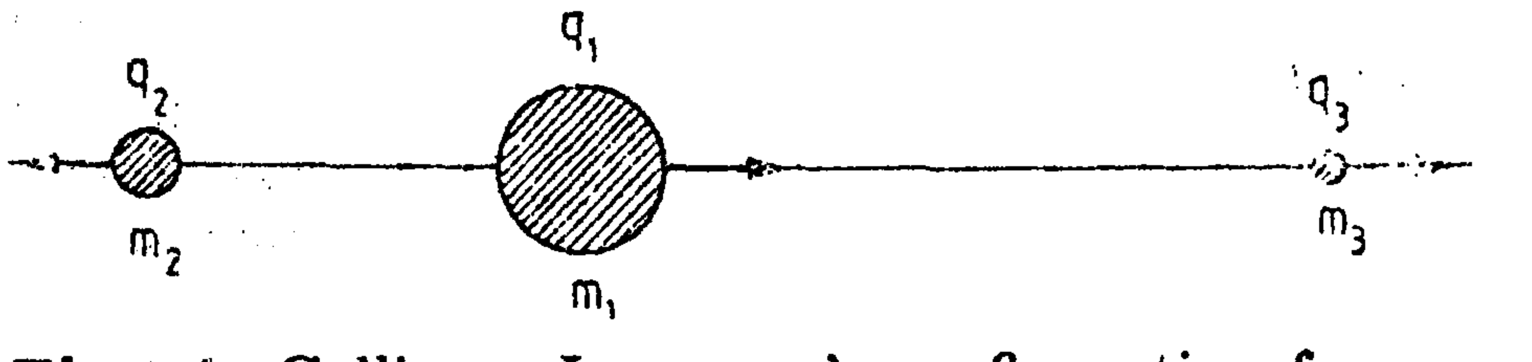

Fig. 4. Collinear Lagrange's configuration for masses or charges.

One can easily find the 'stationary configuration'

$$\Sigma \alpha_i x_i(t) = 0, \quad \alpha - const. \tag{20}$$

by rotating the system around the axis through the origin and equating the centrifugal and centripetal forces: gravitational ($q_i = 0$) or Coulombic ($q_i \neq 0$). Thus one determines constants α_i. For a negative total energy E one obtains in such a way bound state configurations, with three masses rotating like a rigid body (Grujić, 1988; Grujić and Simonović, 1990; see also Grujić, 1982, and Simonović and Grujić, 1987, for alternative methods). For $E > 0$ and all bodies moving away from each other, one has the so-called *fragmentation process*. In the latter case the problem is still beyond the analytical solution, but for a small

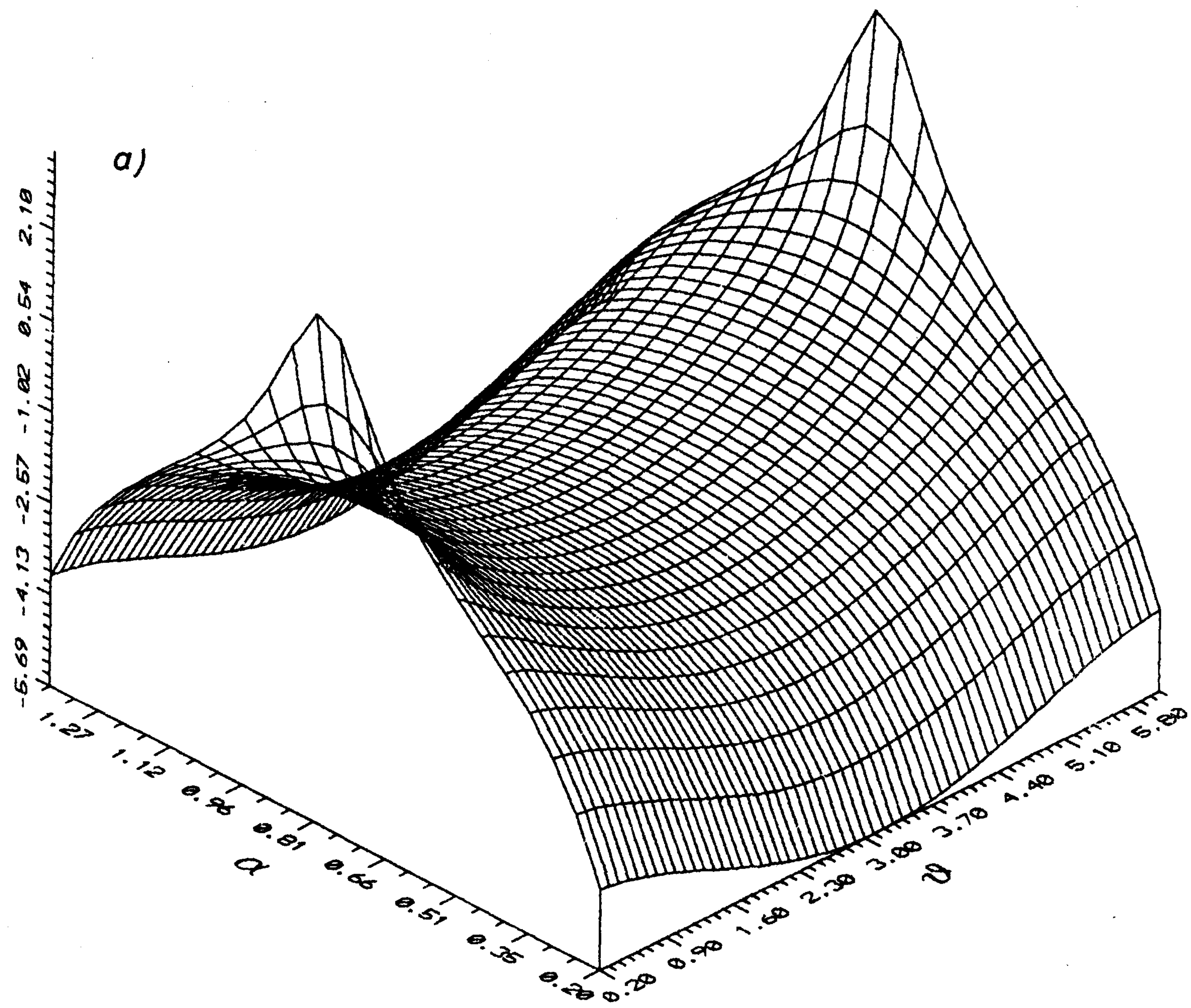

Fig. 5. (**a**) Potential energy surfaces for the electrostatic (Simonović, 1993) three-body systems.

positive E an accurate approximation can be found in the region: $||Ex_i|| << 1$ (in appropriate units; and we shall omit index i)

$$x' = \gamma t^{2/3} \qquad (21)$$

with constant γ independent of energy. That the same kinematics holds for the fragmentation of Coulombic systems and the expanding Universe within Friedman-Robertson-Walker (FRW) cosmological model (*e. g.* Thatcher, 1982) should be no surprise, for in this part of configuration space the motion is determined by the shape of the potential only, which is the same for both forces. If the energy is small receeding bodies must move in a highly correlated manner triple escape to take place. The potential function surface has a saddle point, see Fig. 5, so that motion along the straight line connecting unperturbed bodies (leading configuration) appears unstable, whereas transfersal modes support stable (oscillatory) kinematics.

One can estimate probability for fragmentations of cuch systems, at small energies, by calculating so-called *threshold exponent* K in treshold law

$$P \propto E^K, \quad E \to +0 \qquad (22)$$

K depends on masses for Newtonian and on masses and charges for Coulombic systems. Mention should be made, however, that fragmentation processes are very rare in celestial mechanics, whereas they are important in laboratory – and astro – physical systems, like ionized gasses and stellar atmospheres. Near-threshold processes are important for chemical reaction too, but there they are governed by different type of forces, with potential asymptotics of the form r^{-k}, $k \geq 6$. It is interesting, however, that the same formalism furnish threshold exponent in (22), which will depend, additionally on k (Grujić and Simonović, 1988). This makes power-law forces a wider class of interactions with a specific three-body behaviour energies.

As is well known general solution for three-body problem is still beyond our mathematical tools. One resorts, therefore, to specific configurations, endowed with particular symmetries. These symmetries imply, in their turn, specific correlations between moving objects, what greatly simplifies mathematical analysis. Another type of tractable systems is that of a third body moving essentially at great distance from the rest of the system, so that the perturbation theory applies. We shall call the first class *correlative*

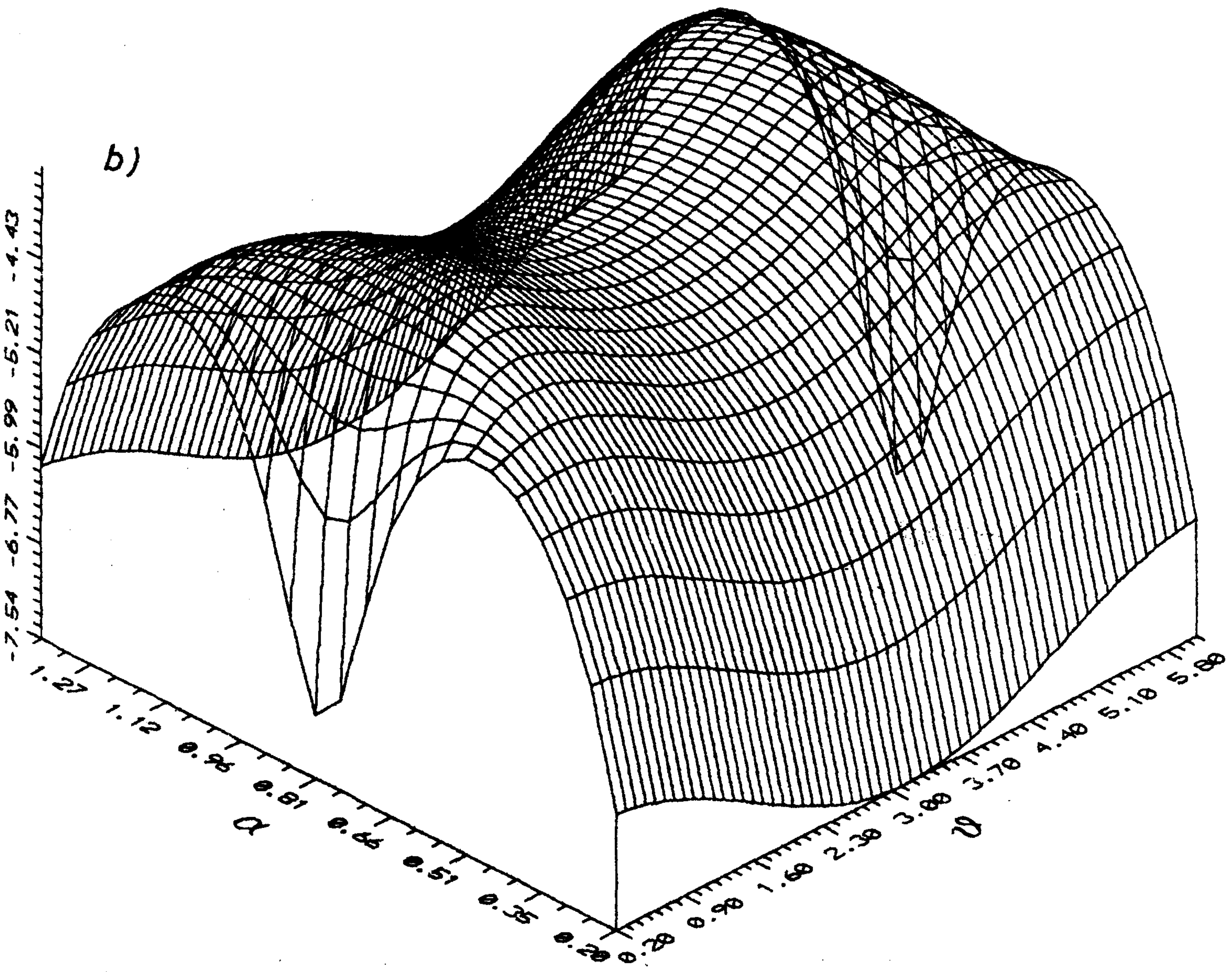

Fig. 5. (b) Potential energy surfaces for the gravitating (Grujić and Simonović, 1990) three-body systems.

and the second *pereturbative* systems, respectively. Of course, gravitating systems possess another class of configuration, planetary ones, which is absent from the atomic realm, due to strong interelectron interaction. Yet, one may define planetary atoms, too, with specific features, as we shall se later on. Now, we shall expound a number of common features of both kinds of systems, for the collinear Lagrange configurations.

(a) *Coulombic case.*

In Fig. 6. we show a general plane configuration for a Coulombic system, with particle 2 and 3 of opposite charge with respect to particle 1 (Grujić, 1988). All three particles move along Keplerian orbits around the common centre of mass, which coincides with a common focus. We shall confine ourselves to the simplest case of the (common) eccentricity of ellipses $\varepsilon = 0$ (circular orbits). If the particles are slightly displaced from the equilibriom positions, they may follow closed (in the rotating frame of reference) orbits around equilibrium positions. We shall restrict ourselves to those perturbation orbits which lie in planes perpendicular to the common axis, as shown in Fig. 7 for the system $[He^{2+} + \pi^{+} + \mu^{-}]$. All particles them move along circles, wing particles in the opposite sense with respect to that of the middle body. In the zero approximation the system forms

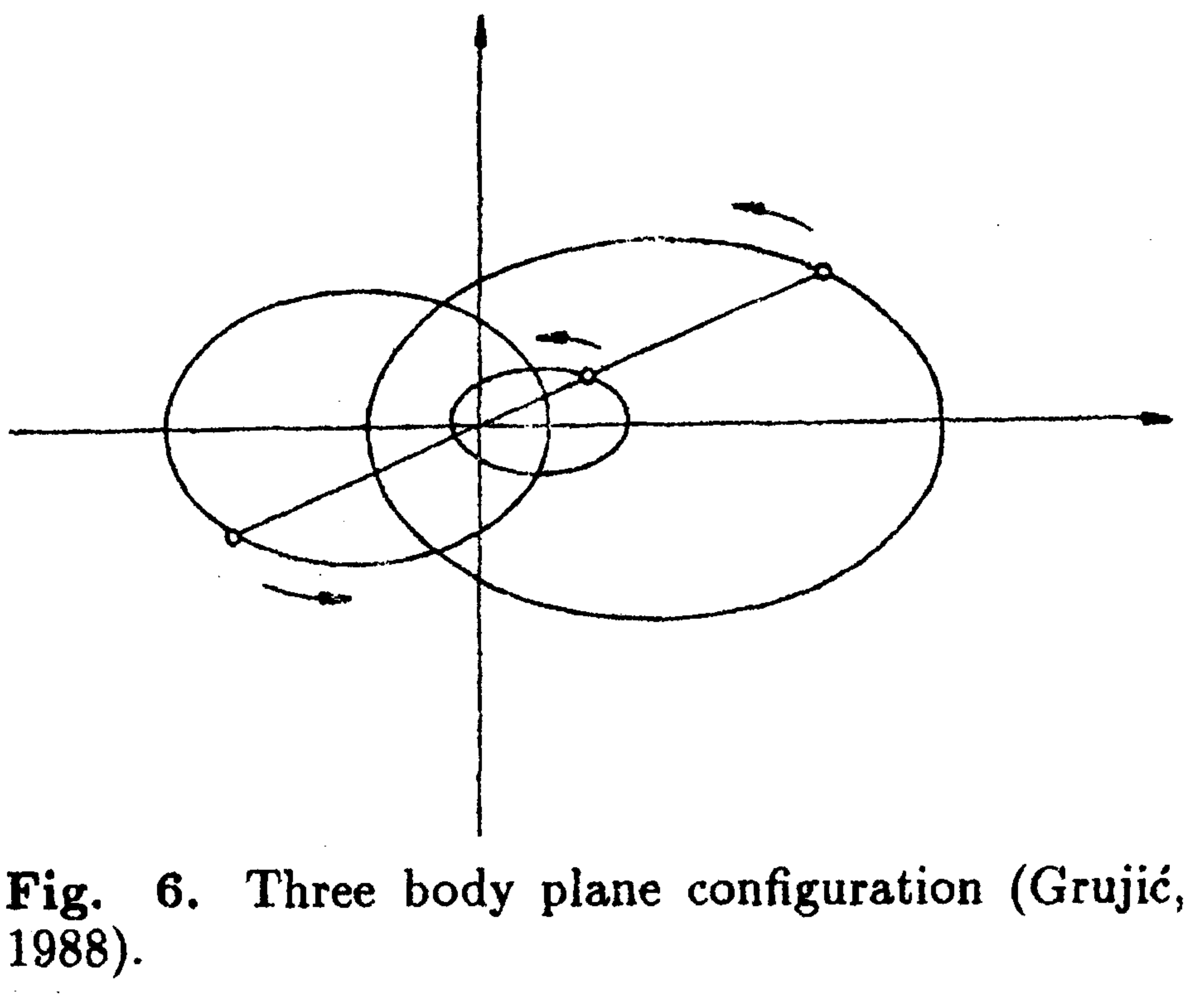

Fig. 6. Three body plane configuration (Grujić, 1988).

a rigid body, rotating around two axes. The system may be treated within Newtonian dynamics, but here one should mention that according to quantum mechanical formalism the circular motion around the rotating axis comes from the superposition of two linear oscillatory motions, mutually perpendicular (that is, restriction to a linear oscillation is not possible for this axially symmetric system). For that

Table I Some characteristic parameters for three-body Coulombic systems (see text) (Grujić, 1988).

Number	System	ω(au)	ω/Ω	K	β
1	$\mu^+ + e^- + \mu^+$	$2.188 \cdot 10^{+4}$	23.51	23.28	1.0102
2	$e^- + \mu^+ + e^-$	5.221	1.160	1.131	1.0253
3	$p + \mu^- + p$	$4.132 \cdot 10^{+3}$	5.000	4.806	1.0404
4	$e^- + e^+ + e^-$	9	2	1.887	1.0601
5	$He^{2+} + \pi^- + \mu^+$	2.241	2.246	2.66	1.1843

reason one talks about rovibronic motion, as in case of triatomic molecules, for instance. If one designates angular frequency around the fixed (laboratory) axis Ω, and that around rotating axis ω, calculations reveal that the ratio ω/Ω assumes values close to those of threshold exponent K from (22), as numerical results in Table I show. One thus arrives at an (approximate) characteristic constant for three-body Coulombic systems $\beta = (\omega/\Omega)/K$, close to one. As can be seen from Table I the gratest β value belongs to the (asyymetrical) system 5. Also one notices comensurability of both frequencies for three equal masses and (absolute value) charges particles (system 4), what implies closed orbits (within approximation used).

Table II Characteristic parameters for symmetrical Newtonian systems (see text).

η	0	1	2.53	3	∞
ω'/Ω	1	1.549	1.9267	2	2.828
ω''	1	1.630	2	2.0716	2.883
$(\omega'/\Omega)/K$	1	1.053	1.0601	1.0604	1.057
$(\omega''/\Omega)/K$	1	1.108	1.1004	1.0984	1.077

(b) *Gravitating systems*

We treat the simple case of symmetrical system, with outer (wing) bodies masses equal ($m_2 = m_3$) (Grujić and Simonović, 1990), and define ratio $\eta \equiv m_2/m_1$. One obtains two frequencies around (rotating) equilibrium points: in the plane of rotation (ω'') and perpendicular to the plane (ω'). In the plane wing bodies execute small elliptical orbits. In Table II we present numerical results for a number of mass ratios. Comensurate "in plane" and Ω frequencies appear at $\eta = 2.53$, but no truly periodic motion (reentrant orbits) is possible for finite masses. As can be seen from Table II, however, this is almost the case for $2.5 < \eta < 3$, and exactly at $\eta = \infty$ (restricted three body problem, *e. g.* Bruno, 1990). In any case calculations shows that frequency ratio is a slowly varying function of η.

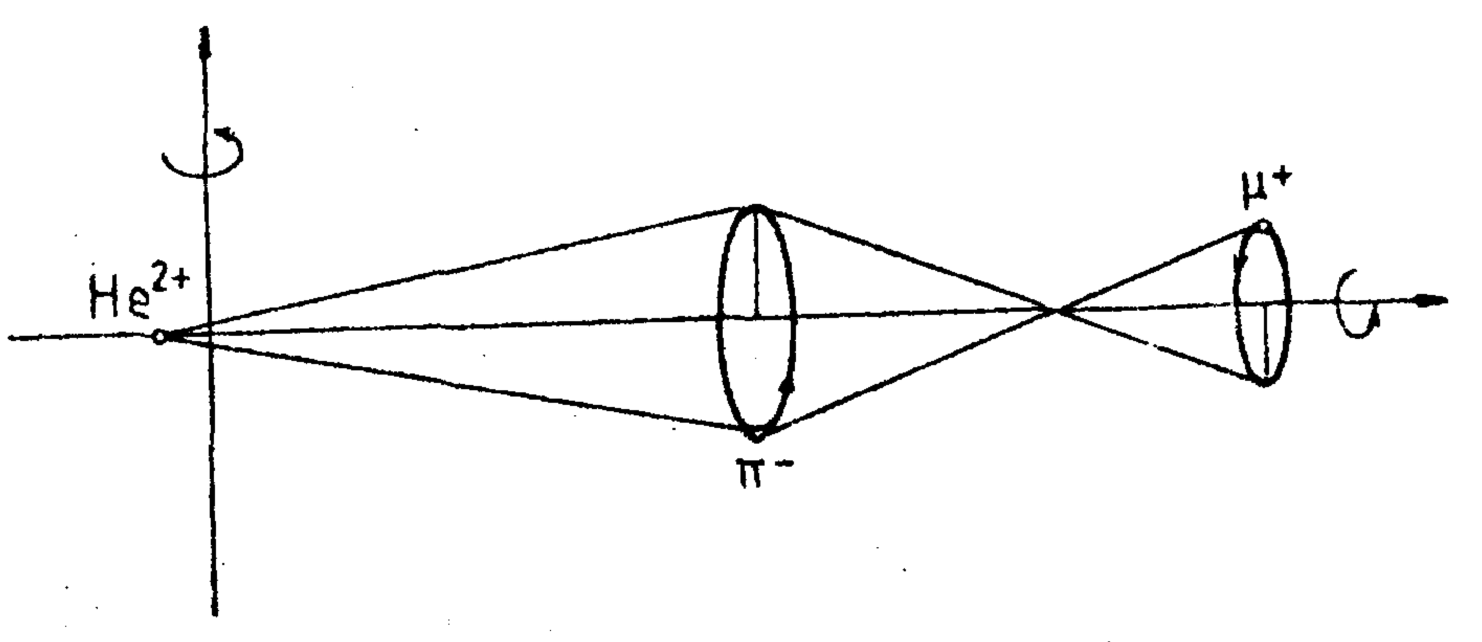

Fig. 7. Three-body Coulombic (quasi)collinear system (Grujić 1988).

Configuration like that in Fig. 7 have been used successfully for describing doubly excited two-electron systems. The question arises, however, if there exist gravitating systems with configurations like that. In principle, a planet of our Solar system might have a twin-sister, rotating for 180° out of phase around Sun, and thus be invisible from the oppositly situated planet. Such a twin-Earth used to be imagined in science fiction (Planet X, see *e. g.* Goldstein, 1981). Considering small eccentricity of our Earth the formalism of accounting small perturbations, as exposed above, could be used to estimate semiaxies of ellipses around equilibrium points, which still prevent phantom-planet from being observed from Earth. Space travels have made such investigations unnecessary, of course.

(ii) *Plane and three-dimensional configurations.*

With respect to Newtonian - Coulombic system parallel nonlinear bound systems appear much more interesting. Unlike two-body systems where difference in interaction was entirely in strength of forces, three-body systems manifest truly an essential difference between Coulombic dichotomy and Newtonian all-attractive interaction. The appearance of repulsive interaction precludes a class of bounded configurations of Coulombic systems for a plane-restricted kinematics, as compared with gravitating ones. The situation appears opposite for a number of out-of-plane motion. This is best illustrated for "rigid-body configurations" $r_{ij} = const$, $i,j = 1,2,3$. Both kind of systems may have stationary configurations by rotating around axis through the centre-of- mass, perpendicular to the plane of configuration (Lagrange configurations), but Coulombic systems have Lagrangian triangle degenerate into straight line, because of two-body repulsion, which forces two of three bodies to locate on the opposite sides of the third one. Nothing of this sort happens to gravitating systems and a (small mass) third body may safely place itself at a (stable) Lagrangian point, out of line connecting two other (massive) bodies (like Troyans asteroides, for instance). On the other hand Coulombic system may have additional rotation axis in the plane of the configuration, like the so called rotor-like Langmuir's helium model. We shall come to this later on.

In the case of one massive particle and two electrons, like system 2 in Table I, one speaks of helium-like atoms (ions). Figure 5 corresponds then to the so-called synchronous (mode) configuration or kinematics, with particle 1 (α – particle, for instance) at the origin (centre-of-mass) and two electrons moving around *in phase* (that is out-of-phase for 180°). This is obviously just a special case of the general collinear configurations, considered above. There are, however, other types of kinematics, with specific symmetries, which ensure periodic, or reentrant orbits. One of them has been studied recently (Grujić and Simonović, 1991) and we shall expound it briefly here.

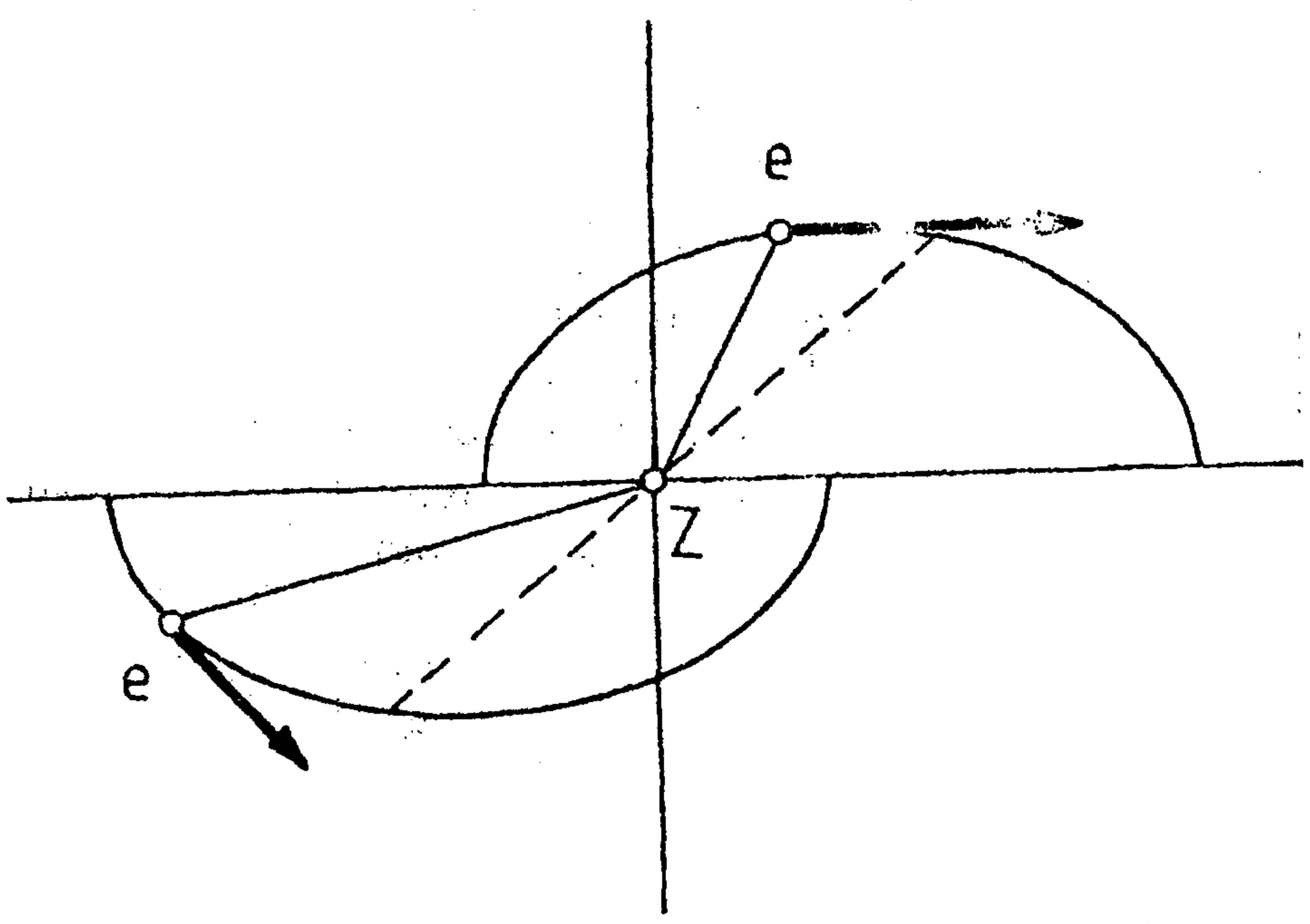

Fig. 8. An asynchronous (mode) two-electron system (Grujić and Simonović, 1991).

This is so-called asynchronous (mode) model, where electrons move out-of-phase, with one particle at perihelion while the other is at aphelion, and vice versa (see Fig. 8). It is believed these configurations appear as doubly excited-helium atoms (or helium-like ions). One can evaluate (descrete) spectra of such a configuration and compare that with standard synchronous (so-called Bohr-Sommerfeld) model (Grujić and Simonović, 1993). It is interesting that very odd configurations may arise within the asynchronous model, with particle orbits very different from Keplerian ones (Simonović, 1993).

As for the gravitating systems no strict analogy with Coulombic case appears possible. The only way to fix the central body at the origin is to put its mass very large (in principle infinite). But then, no correlations between light bodies arise and one may, by a suitable choice of initial conditions, arrange any sort of kinematics, including that of the asynchronous helium model. On the other hand, if one deals with finite-mass (e. g. equal-mass) systems, a more complicated kinematics would develope, with central body moving out from the centre-of-mass. In that case, however, the analogy with atomic case would be lost, due to the equality of inertial and gravitational masses. Anyway, as far as we know, no such calculations have been carried out for gravitating systems. On the other hand, complicated orbits may be found within the synchronous mode *see Fig. 5 in Broucke et al.*, 1981).

Another interesting class of plane configurations has been investigated by Broucke *et al.* (1981). These arise, in fact, from the collinear configurations, but with large amplitude deviations from the basic configurations. However, when viewed from the laboratory reference system, they reveal an unusual type of correlated motion, as shown in Fig. 9.

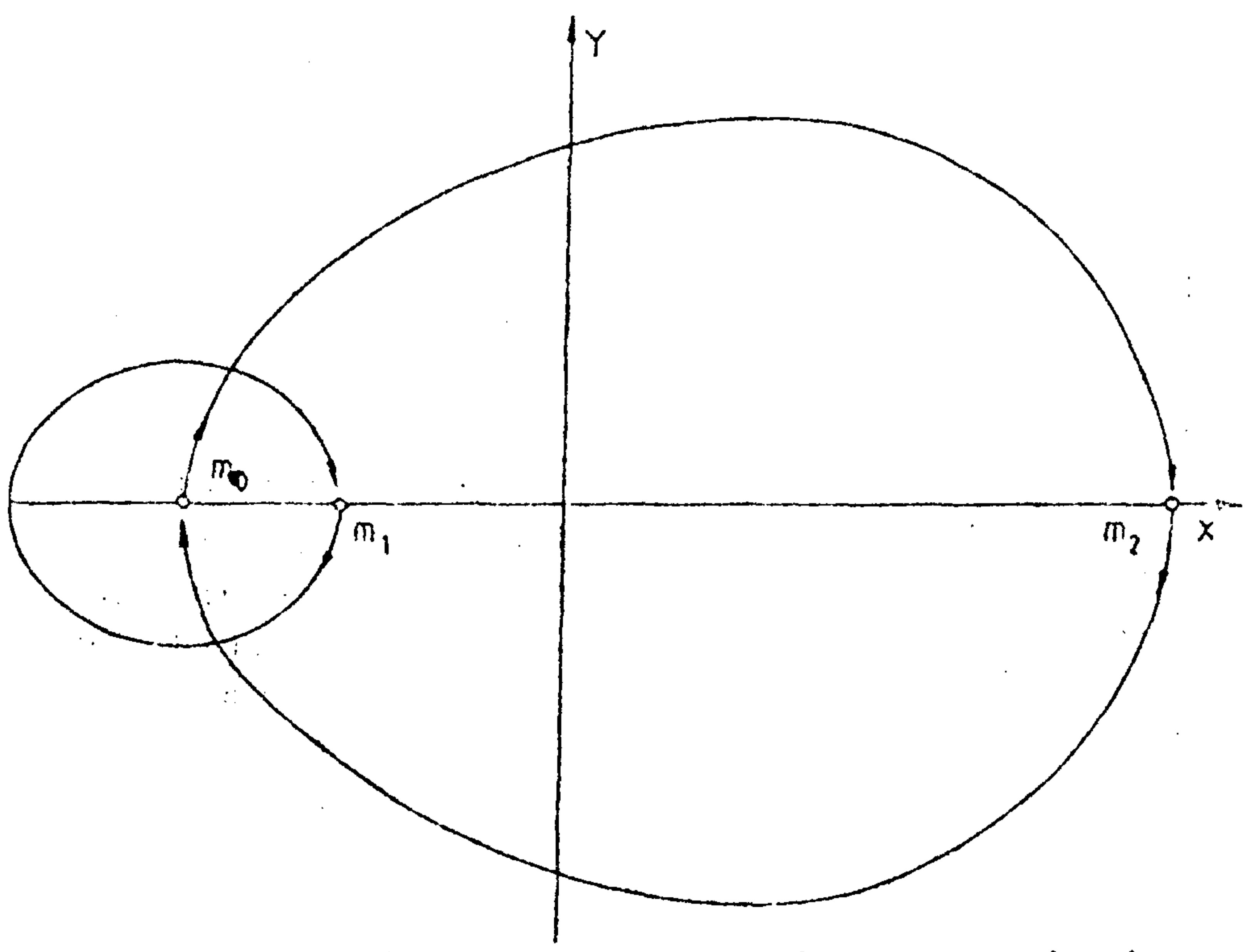

Fig. 9. Reentrant orbits for equal mass gravitating three-body systems (Broucke *et al.*, 1981).

In principle, this type of correlated motion may be found in other types of interactions, including that of Coulombic one. Some calculations along these lines are in progress.

Finally we come back to the Langmuir's helium model (see Fig. 10), the only truly three-dimensional configuration we have considered so far. As in the case of (quasi)collinear case, it is possible to carry out an analysis of small deviations from the basic (skeleton) configuration (Dimitrijević and Grujić, 1984) and evaluate the complete rovibronic energy

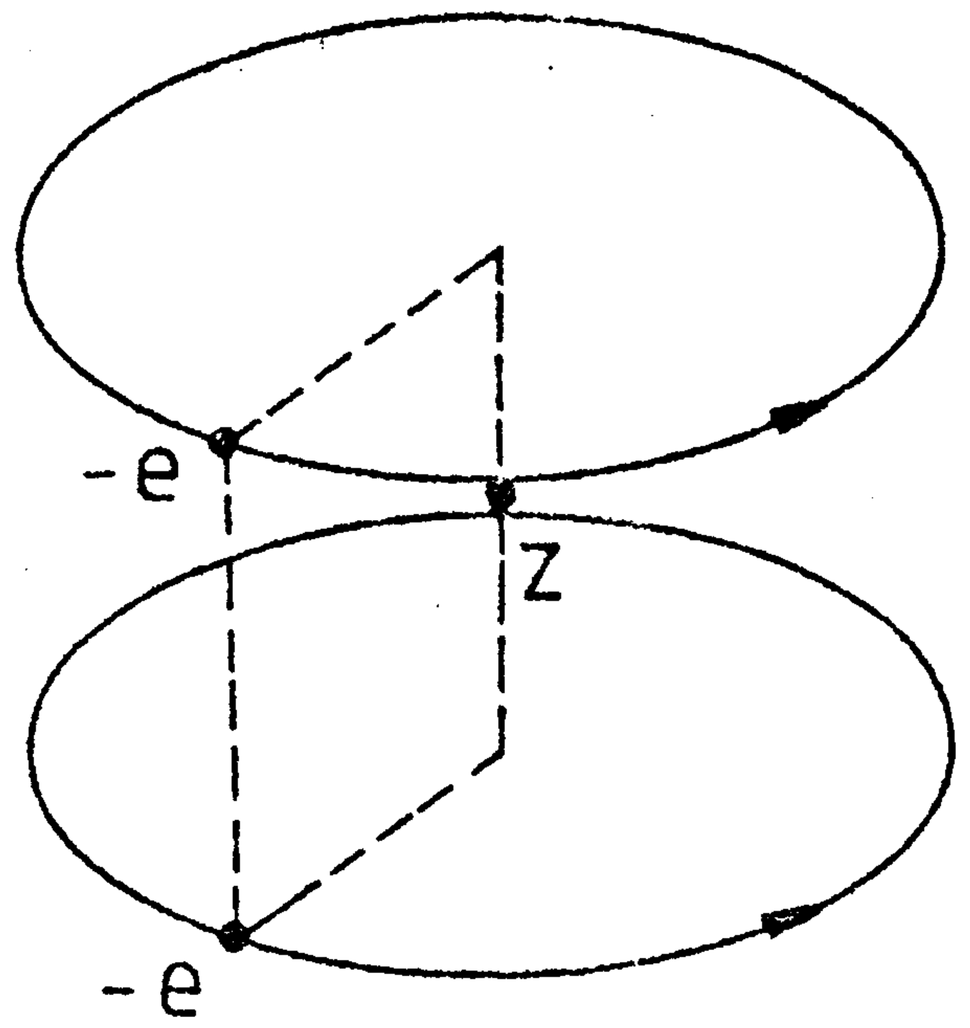

Fig. 10. (a) Langmuir's model for helium atom.

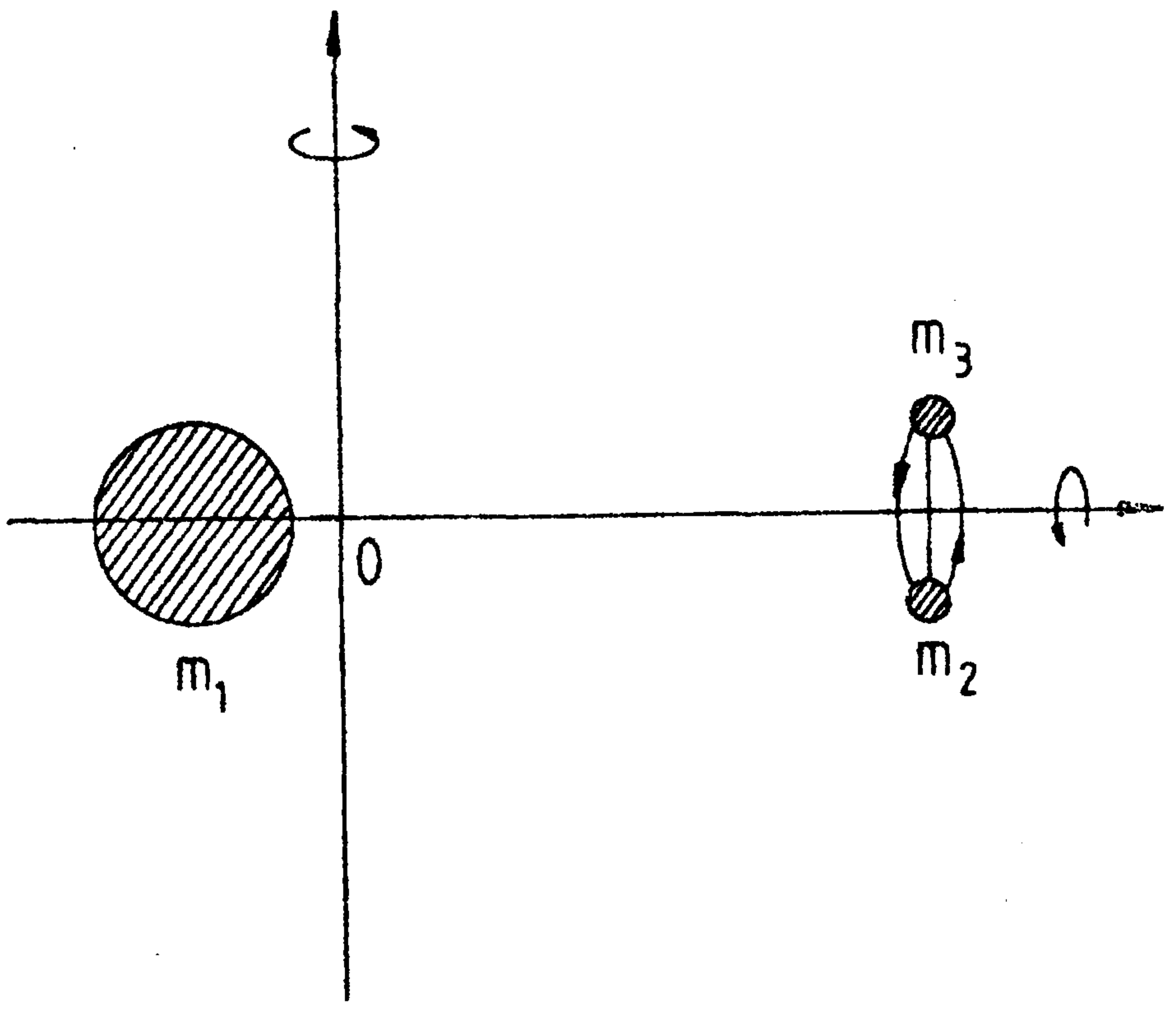

Fig. 10. (b) an analogous model for gravitating bodies.

spectre. It is interesting to note that recently an odd effect has been discovered for systems with Langmuir-like configurations. Namely, Poirier (1989) has found that for a model system with the heavy charge $0.459 < Z < 1$ (instead of α – particle) unsymmetrical configurations appear possible, with electrons differently displaced from the origin. This phenomenon turns out to be realistic one, for systems like $[I^- + \alpha + \alpha]$, when the finite-mass effects are accounted for (Grujić and Simonović, to be published). In the case of gravitating systems, bodies circling around the third, massive one, cannot remain at rest in their own body-fixed frame, because of the absence of mutual repulsion. The stationary equilibrium may be achieved, however, by making the two-body subsystem rotate about their own (rotating) axis, as shown in Fig. 10(b). If the closed orbit (as viewed from the comoving reference system) is sufficiently short, (compared with the imagined orbit of the centre-of-mass), it may be approximated with a circle and the entire three-body system can be considered to be a rigid-body structure, with two axes of rotation, one fixed in space and the other rotating about the first.

It is worth mentioning, within the present context, that the well known Russian mathematician, physicist and astronomer A. Friedman, was engaged at a time in constructing a three-dimensional helium model (see, *e. g.* Efremidze and Frenkel, 1989), which in modern terminology would correspond to the planetary atoms, with the outer electron moving along a circle and the inner one following another small circular orbit, in a plane (on the other side of the nucleus) perpendicular to the first electron orbit plane. This model has been abandoned, as were many other proposed at the dawn of Quantum mechanics, but unlike these latter, has remained largely unknown to the wider physical community. As far as we know no attempt to investigate this model for gravitating systems has been made so far.

Though gravitational effects does not show up in the realm of atoms, where Coulomb force dominate, mass-effects may be very prominent in determining bound-state properties and collision processes features. The latter are evident from Table I (threshold exponents in rows 1 and 2, for instence). Masses (better to say mass ratios) influence in a decisive way whether a three-body (Coulombic) bound state is possible or not. Thus for the case of systems with unit charges it turns out that if a bound state of a charge in the force-field of a binary system is to be formed (this type of system corresponds to the restricted three-body problem in celestial dynamics), the mass-ratio for the two-body system must not exceed a threshold value (Frolov and Bishop, 1992). This is explained by the (in)sufficient polarization force, induced by the third particle. Only if one of the binary-component is enough small, third body can induce a sufficiently strong dipole moment of the binary, so that a binding potential appears. This situation is just the opposite what one would expect in purely gravitating systems, where within the restricted three-body systems equal-mass binary binds too.

Polarization force is present in ion – atom interactions. Particularly it appears important for small-energy electron – atom collisions, or Rydberg states of atomic systems. It may be visualized as a deformation of the electron cloud around the nucleus, by the Coulomb force of the impinging, or excited particle. Although polarization potential rarely binds, it plays vital role in the so-called resonant effects and exerts a noticeable influence on the energy spectrum of Rydberg states.

Nothing of this sort appears with gravitating planetary systems, like our Solar one. The closest analogy would be an interaction of a small satelite with a planet endowed with a ring, like Saturn. But the mass ratio of Saturn, for instance, to the ring mass appears so large, that an influence of the deformation of a ring due to the satelite presence is negligible. On the other hand deformations due to gradient of the gravitational force do play crucial role in the stellar binaries, as the so-called Roche lobes (e. g. Martinov, 1965) testify. An atomic analogy with (fluid) mass transfer through the inner Lagrange point from one to the other binary would be so-called charge transfer. This occurs, for instance, between two oppositly charged ions, like H^- and p

$$p + H^- \rightarrow H + H \tag{23}$$

There is, however, one important difference, as to the mechanism which underlies the exchange of electrons and neutral (fluid) mass. The first is governed by quantum mechanical discrete change of states, with one of electrons in the (23) jumping to the proton, while the remaining atomic electron goes down to the hydrogen ground state. The mass transfer from one to the other Roche lobe, is described, on the other hand, by the classical (discrete and fluid) mechanics, which prescribes a continuous flow.

More close analogy appear between a planet capturing in passing a satelite from another planet and an electron capture by an ion from a Rydberg atom. Both processes may be described by the classical dynamics, in the atomic case at least in an approximate manner.

5. CONCLUDING REMARKS

As we have tried to show, under normal circumstances gravitational and electrostatic system display a number of common features, but also some essential differences. These latter mainly come from a few peculiar properties of gravitational force; (a) the unique gravitational charge. (b) equality of gravitational charge and inertial mass; (c) extremely small strength of the gravitational force. The first property precludes gravitation from taking part in shaping the structure in the microworld domain, whereas the third excludes competition between gravitation and other forces, in particular with Coulombic one.

Within the (quasi)two-body systems both types of interactions provide similar structures, but with an increase of number of constituents, disimilarities become more prominent. Nevertheless, many formal common features enable one to make use of formalism developed for one type of interaction in treating systems with the other force. Moreover, a number of features displayed by one type of systems, often very peculiar, may be expected, *mutatis mutandis*, in other systems governed by different force.

Of course, as mentioned above, there are situations where classical and Newtonian formalisms fail. These are atomic systems with small quantum numbers, where Schrödinger theory applies, from the one side, and strong gravitational fields, when Einsteinian theory appears indispensable, from the other. The domain of ultrastrong gravitational fields, where presumably a hypothetical theory of quantum gravity must be applied and where the present sort of parallel would become superfluous, is out of scope of the present overview.

The principal aim of this review, besides a number of heuristic implications revealed, is to draw attention of researchers in both fields, atomic physics and celestial mechanics and astronomy, to methods used in the other field that could be of interest to their own research. If we have succeeded in making those researchers aware of these possibilities, we would feel our task worth trying.

Acknowledgments – The ideas expounded in this paper have developed during my almost thirty years involment in atomic physics calculations and interest in astronomy and astrophysics. I would like to thank Dr Nenad Simonović for useful discussions and for the help in preparing the manuscript, in particular for supplying figures 5. We are indebted also to Mg Aleksandar Tomić for providing us with his unpublished results and helpful discussions on them.

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ЊУТНОВСКИ И КУЛОНОВСКИ СИСТЕМИ

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Направљена је компаративна анализа система са малим бројем конституената са гравитационом и електромагнетном интеракцијом, са нагласком на сличностима и разликама релевантних карактеристичних особина. Приказан је скорашњи напредак у изучавању вишеструко побуђених атома, као и планетарних небеских система и дискутоване су неке интересантне сличности.