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THERMAL PROCESSES AND SPIRAL DENSITY WAVES II. STABILITY OF THE GASEOUS DISC OF THE GALAXY

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SUMMARY: The gaseous component of the galactic disc with heating and cooling processes (without heat conduction) is considered. The viscous and magnetic forces are not taken into account. Using the infinitesimally-thin-disc approximation one derives the dispersion equation for the case of small spiral perturbations and one finds its approximative solution. A criterion for the disc stability in the presence of thermal processes is also derived.

1. INTRODUCTION

In Paper I (Angelov, 1992) one considers the response of the nonviscous, magnetic-field free, galactic disc to a spiral perturbation in the gravitational field. Using the infinitesimally-thin-disc approximation one introduces in the energy equation the heating-cooling function $\mathcal{L}(\sigma, T)$ without heat conduction. For the case of an m-spiral perturbation

$$f'(R,\varphi,t) = \widehat{f}(R)e^{i(\omega t - m\varphi + \int k(R)dR)} \tag{1}$$

with $|\int kdR| \gg 1$, one obtains a system of linear equations

$$i\overline{\nu}\widehat{\sigma} + \frac{ik\sigma^0}{\kappa}\widehat{v}_R = 0, \tag{2}$$

$$\frac{ik}{\kappa\sigma^0}c_s^2 h\widehat{\sigma} + i\overline{\nu}\widehat{v}_R - \frac{2\Omega}{\kappa}\widehat{v}_\varphi = \frac{ik\widehat{\Phi}}{\kappa}, \tag{3}$$

$$\frac{\kappa}{2\Omega}\widehat{v}_R + i\overline{\nu}\widehat{v}_\varphi = 0. \tag{4}$$

In (1)-(4) the designations are usual, exept

$$h = \frac{\frac{1}{\gamma}(x_T - x_\sigma) + i\overline{\nu}}{x_T + i\overline{\nu}}, \tag{5a}$$

where

$$x_T = \frac{(\gamma-1)\mu\mathcal{L}_T}{\kappa\mathcal{R}_g}, \quad x_\sigma = \frac{(\gamma-1)\mu\sigma^0\mathcal{L}_\sigma}{\kappa T^0\mathcal{R}_g}. \tag{5b}$$

Here, $\mathcal{L}_\sigma$ and $\mathcal{L}_T$ are the partial derivatives of $\mathcal{L}$ in σ and T, respectively, whereas $\overline{\nu}$ is equal to $\frac{\omega - m\Omega}{\kappa}$.

The subject of the present paper is the galactic-disc stability in the presence of thermal processes. The thermal instability of a nonviscous medium with no volume forces was considered by Field(1965), the character of the viscous-dissipation influence by Angelov(1989) and the gravitation effects on the thermal instability by several authors (e.g. Marochnik, Suchkov, 1984).

2. THE DISPERSION EQUATION AND STABILITY CRITERION

The asymptotic solution for the function of the gravitational potential Φ in the symmetry plane of the disc (Lin, Shu, 1964), where

$$\widehat{\Phi} = \frac{2\pi G}{|k|}\widehat{\sigma},$$

will be used. By substituting $\widehat{\Phi}$ together with h from (5a) in (3), System (2)-(4) becomes homogeneous with $\widehat{\sigma}, \widehat{v}_R, \widehat{v}_\varphi$ as the unknowns. The condition for obtaining nontrivial solutions (system determinant equal to zero) yields the dimensionless dispersion equation for small spiral perturbations

$$(\overline{\nu}^2 - \nu^2)(\overline{\nu} - ix_T) - iA = 0, \quad \overline{\nu} \neq 0. \qquad (6)$$

Here one has

$$A = x^2\frac{(\gamma-1)x_T + x_\sigma}{\gamma}, \quad x^2 = \frac{k^2c_s^2}{\kappa^2}, \qquad (7)$$

and $\nu \in Re$ satisfies the equation

$$\nu^2 = 1 - |k|_* + \frac{Q^2}{4}k_*^2 \qquad (8)$$

with

$$Q^2 = 4\frac{k_c^2c_s^2}{\kappa^2}, \quad |k|_* = \frac{|k|}{k_c} \quad and \quad k_c = \frac{\kappa^2}{2\pi G\sigma^0}$$

(k_c is the Jeans wave number).

For a given basic state and fixed $k \in Re$ the dispersion equation is solved for the unknown frequency. With regard that

$$\overline{\nu} = u + iw,$$

$$where \quad u = \frac{Re(\omega) - m\Omega}{\kappa} \quad and \quad w = \frac{Im(\omega)}{\kappa},$$

one obtains for the real and imaginary parts of Equation (6)

$$u^2 - w^2 - \nu^2 - 2w(w - x_T) = 0, \quad u \neq 0, \qquad (9a)$$

and

$$(w - x_T)(u^2 - w^2 - \nu^2) + 2u^2w - A = 0, \qquad (9b)$$

respectively. If $A \equiv 0$, from (9a) and (9b) it follows $w = 0$, $u^2 = \nu^2 \neq 0$, and (6) is reduced to the already known dispersion equation (8) without thermal processes ($\mathcal{L} = 0$): in the case of an axially symmetric perturbation ($m = 0$) the stability criterion for all waves is $Q^2 \geq 1$. By including the thermal processes the wave frequency ω becomes a complex quantity and the stable states of the disc are determined by $w > 0$. The equation for w to be analysed follows from (9a) and (9b): by eliminating u^2 one obtains

$$2w[(2w - x_T)^2 + \nu^2] - A = 0. \qquad (10)$$

In the present paper only purely real and positive solutions of this equation are of interest. They will be looked for by applying the standard methods or, simply, as the abscissae of the points common to the function

$$F(w) = w[w^2 - x_Tw + \frac{1}{4}(x_T^2 + \nu^2)]$$

and the straight line $D = \frac{A}{8} = const$. The analysis of the roots and extrema $F(w)$ yields one or three such points (as could be expected considering that (10) is a cubic equation with real parameters ν^2, x_T, A). In addition there is one, and only one, unequivocally defined common point in the domain $w > 0$, when simultaneosly $D > 0$ and $x_T^2 \leq 3\nu^2$.

The condition $D > 0$, i.e. $A > 0$ on the basis of (7), yields

$$(\gamma - 1)x_T + x_\sigma > 0 \qquad (11)$$

or, with dimensionful variables in view of (5b),

$$(\gamma - 1)T^0\mathcal{L}_T + \sigma^0\mathcal{L}_\sigma > 0$$

(γ in the disc plane). Therefore, Inequality (11) is a "planar approximation" for the thermal-stability criterion of the wave mode with no heat-conduction and viscous-friction terms (Field, 1965; Angelov, 1989).

The condition $x_T^2 \leq 3\nu^2$ with regard to (8) yields

$$Q^2 \geq 4|l|\{1 - (1 - \frac{x_T^2}{3})|l|\}, \qquad (12)$$

with $|l| = 1/|k|_* = |\lambda|/\lambda_c$, where $\lambda_c = 4\pi^2G\sigma^0/\kappa^2$ is the Jeans wavelength. Relation (12) relates the parameters Q^2 and x_T^2 over a given length of stable waves, i.e. it can be treated as a condition for Q^2 in the stability domain with $x_T^2 < 3$ given. The dependence $Q^2(|l|, x_T^2)$ in the limiting case (equality in (12)) is presented in Fig.1. The region between the $|l|$-axis and the curve with $|x_T|$ given belongs to the unstable waves, while the outer region, up to $x_T^2 = 3$, together with the curve belongs to the stable ones. The curve with $x_T = 0$ determines the marginal stability of the disc with respect to the axially symmetric perturbations in the absence of thermal processes. As seen, each value $|x_T| < \sqrt{3}$ generates a new

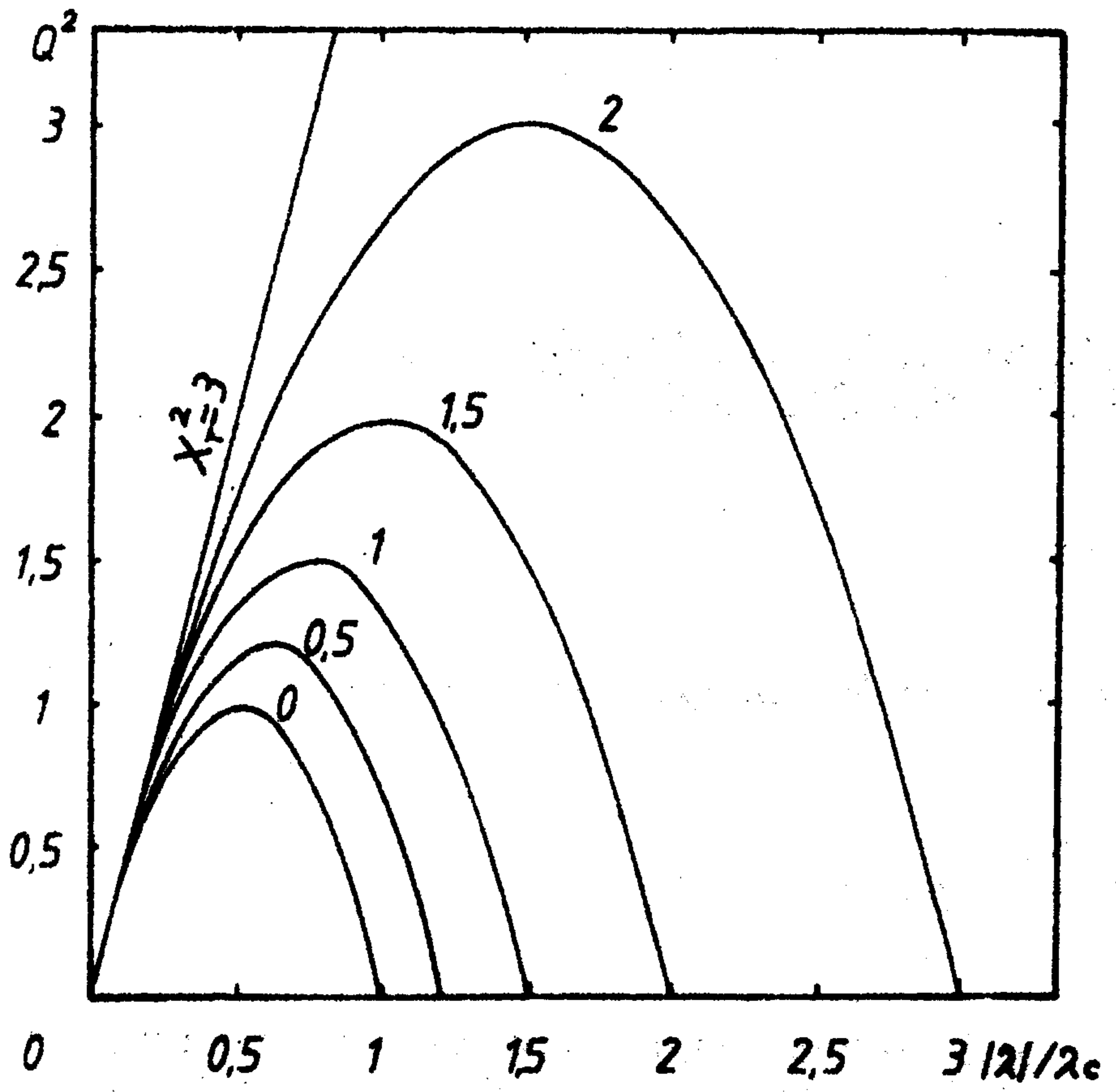

Fig. 1. The marginal stability of the disc with thermal processes for a few values of x_T^2.

marginal curve with $|l|_{max} = (1 - x_T^2/3)^{-1}$ and the abscissa of the maximum at $|l|_{max}/2$. Compared to the curve with $\mathcal{L} = 0$, the maxima are shifted upwards and towards higher $|l|$. For a given $x_T^2 < 3$ the stability of the long and short waves of both types (l and t) is achieved provided that

$$Q^2 \geq \frac{1}{1 - x_T^2/3}. \qquad (13)$$

In any case it is fulfilled $Q^2 > 1$ being the superstability condition for the disc with $\mathcal{L} = 0$. Condition (13), together with (11), appears as a stability criterion for the disc with thermal processes.

At the end an approximative solution of the dispersion equation (6) is presented. By expanding the solution in a series in small parameters $|x_T|, |x_\sigma| \sim \epsilon \ll 1$ one obtains

$$\overline{\nu}_k = \nu_k \left[1 + \frac{A}{2\nu^4}\left(\frac{3A}{4\nu^2} - x_T\right)\right] +$$

$$i\frac{A}{2\nu^2}\left[1 - \frac{1}{\nu^2}\left(\frac{A}{\nu^2} - x_T\right)^2\right] + O(\epsilon^4). \qquad (14)$$

Here $k = 1, 2$; A is expression (7), and $\nu_{1,2} = \pm|\nu|$ are the nonzero solutions of Equation (8). In the stability domain of all the waves of type(1) the linear part of $Im(\overline{\nu}_k)$ in (14) indicates $A > 0$(condition (11)) and $\nu^2 > 0$ (8), i.e. $Q^2 > 1$ (13).

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ТОПЛОТНИ ПРОЦЕСИ И СПИРАЛНИ ТАЛАСИ ГУСТИНЕ II. СТАБИЛНОСТ ГАСОВИТОГ ДИСКА ГАЛАКСИЈЕ

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Оригинални научни рад

Разматра се гасна компонента галактичког диска са топлотним процесима (у одсуству топлотног провођења), без вискозних и магнетних сила. У апроксимацији бесконачно танког диска, изводи се дисперзиона једначина малих спиралних поремећаја и даје се њено апроксимативно решење. Изводи се критеријум стабилности диска са топлотним процесима.