

## THE MOTION OF AN ARTIFICIAL SATELLITE IN THE OBLATE ATMOSPHERE

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**SUMMARY:** Under conditions of the TOTAL DENSITY MODEL (Sehnal, 1988) the changes of the satellite orbital elements are concluded (Šegan, 1989) and first and second order theories are constructed. We assume that the constants defining thermospheric density model are small (parameters).

### 1. INTRODUCTION

It is evidently that not exist the general theory which gives the description of the orbital changes for a time interval longer than one revolution. The absence of that is caused by the complexity of the atmospheric density distribution (modeling).

The result of all "aeronomic" models are usable only for the determination of the orbital perturbations merely over one revolution of a satellite around the Earth. The determination of the orbital changes under influence of the atmospheric drag just over one revolution is a common procedure adopted in the present theories.

Using the Sehnal's (1986, 1988a, 1988b) method of approximation of the total thermospheric density distribution and results from our previous paper (Šegan, 1987), we shall develop the theory of second order.

### 2. THE AIMS

By usage computer algebra we were derived general linear expression for the first order perturbations of the satellite orbital elements under influence of a drag:

$$\frac{d\Theta}{dE} = -f_1(a)\delta f_2(e, E)\rho\left(\frac{\sin}{\cos}\right)^k i\left(\frac{\sin}{\cos}\right)^l u, \quad (1)$$

where

$\Theta$  ( a, e, i,  $\Omega$ ,  $\omega$  )

a – semi-major axis

e – eccentricity

E – eccentric anomaly

$\delta$  – drag force per unit mass (satellite dependant)

and from Sehnal's total density model

$$\rho = \sum_{k=1}^K \rho_s^k e^{\frac{r_{p0}-r}{kH}} = \sum_{k=1}^K \sum_{i=1}^I A_i^{(k)} f_i e^{\frac{r_{p0}-r}{kH}}, K = 3$$

or

$$\rho = k_0 f_0 f_x \sum_{n=1}^7 h_n g_n, \quad (2)$$

where  $A_i^{(k)}$  expresses the geometry while  $f_i$  are functions of the physical parameters. By this way we overruled the standard difficulties.

An analytical integration (1) under interval of the one orbital revolution yeald:

$$\Delta\Theta = Nf(a, e), \quad (3)$$

where perturbations and influences of the other orbital elements are neglected.

## THE SECOND ORDER PERTURBATIONS

For the prediction of the long time perturbations ("second order") we need

$$\Delta^1\Theta = Nf(a, e), \quad (3a)$$

and expressions for the 2nd order terms

$$\Delta^2\Theta = \frac{1}{2}N^2(f(a, e)\frac{d}{da}(f(a, e)) + f(a, e)\frac{d}{de}(f(a, e))), \quad (4)$$

and

$$\Delta\Theta = \Delta^1\Theta + \Delta^2\Theta. \quad (5)$$

Since we know the analytical expressions for  $\Delta a_0$  and  $\Delta e_0$ , we can compute easily their derivatives acc. to  $a$  and  $e$  (index 0 is for perigee distance). Also, we have to take into account the derivatives of the Bessel functions, too.

This is the way to construct the theories of higher orders. As a result the evolution of the elements  $\Theta$  will be described generally as a polynomial of the desired order.

The results can be checked on the observed satellite orbital changes. We have the observed values of the semi-major axis and eccentricity of the ANS satellite. We see that the first order theory describe the changes as a linear function of time and does not provide reliable information on the evolution of the orbit whereas the second order theory corresponds to the observed values much better.

In much cases we must take into account the influences of the orbital elements changes in their complexity. It will be object for the our next paper.

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## КРЕТАЊЕ ВЕШТАЧКИХ САТЕЛИТА У ВИСОКОЈ АТМОСФЕРИ

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 Предходно саопштење

Под условима модела тоталне густине (Sehnal, 1988) изведене су промене путањских елемената сателита (Шеган, 1989) и конструисане су теорије

првог и другог реда. Претпостављамо да су параметри који дефинишу модел густине термосфере мали.